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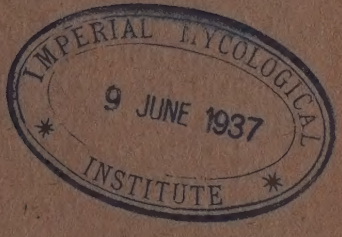

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MATHÉMATIQUES

SUR LA MEILLEURE APPROXIMATION DES FONCTIONS PÉRIODIQUES DÉRIVABLES AU MOYEN DE SOMMES TRIGONOMÉTRIQUES

Par N. ACHYESER et M. KREIN

(Présenté par S. N. Bernstein de l'Académie, le 11. III 1937)

Désignons par $E_{n-1}(F)$ la meilleure approximation de la fonction périodique $F(x)$ * au moyen de sommes trigonométriques d'ordre $\leqslant n-1$.

Soit H_r l'ensemble de toutes les fonctions périodiques * .

$$f(x) = \sum_{k=0}^{\infty} (a_k \cos kx + b_k \sin kx)$$

dont la dérivée d'ordre r : $f^{(r)}(x)$ est de module $\leqslant 1$; soit enfin

$$f^*(x) = \sum_{k=0}^{\infty} (b_k \cos kx - a_k \sin kx).$$

Le but de cette note est de démontrer les relations suivantes:

$$\sup_{f \subset H_r} E_{n-1}(f) = \frac{4}{\pi} \frac{1}{n^r} K_r, \quad K_r = \sum_{\nu=0}^{\infty} \frac{(-1)^{\nu(r-1)}}{(2\nu+1)^{r+1}}, \tag{1}$$

$$\sup_{f \subset H_r} E_{n-1}(f^*) = \frac{4}{\pi} \frac{1}{n^r} K_r^*, \quad K_r^* = \sum_{\nu=0}^{\infty} \frac{(-1)^{\nu r}}{(2\nu+1)^{r+1}}. \tag{2}$$

Ces relations précisent les théorèmes bien connus de M. Jackson.

Remarquons d'abord que la fonction

$$f_0^{(r)}(x) = \frac{4}{\pi n^r} \sum_{k=0}^{\infty} \frac{\sin\left(\overline{2k+1}\, nx - \frac{\pi}{2} r\right)}{(2k+1)^r}$$

appartient à H_r, car

$$f_0^{(r)}(x) = \frac{4}{\pi} \sum_{k=0}^{\infty} \frac{\sin(2k+1)nx}{2k+1} = \operatorname{sgn}(\sin nx).$$

* En général complexe.

D'ailleurs il est aisé de montrer que pour chaque r la fonction $f_0(x)$ prend la valeur $\pm\frac{4}{\pi n^r}K_r$ en changeant de signes alternativement en $2n$ points:

$$x_\lambda = \frac{\varepsilon\pi}{2n} + \frac{\lambda\pi}{n} \qquad (\lambda = 0,\ 1,\ \ldots,\ 2n-1),$$

(où $\varepsilon = 0$ ou $= 1$ suivant que r soit impair ou pair) et le maximum de sa valeur absolue est égal à $\frac{4}{\pi n^r}K_r$.

Il en résulte d'après le théorème de Tchebicheff

$$E_{n-1}(f_0) = \frac{4}{\pi}\frac{1}{n^r}K_r.$$

D'une manière analogue, en posant

$$f_0^*(x) = \frac{4}{\pi n^r}\sum_{k=0}^{\infty}\frac{\cos\left(2k+1\,nx - \frac{\pi}{2}r\right)}{(2k+1)^r},$$

on trouve que

$$E_{n-1}(f_0^*) = \frac{4}{\pi n^r}K_r^*.$$

Il nous reste à montrer que pour toute fonction $f(x) \subset H_r^2$

$$E_{n-1}(f) \leqslant \frac{4}{\pi}\frac{1}{n^r}K_r, \quad E_{n-1}(f^*) \leqslant \frac{4}{\pi}\frac{1}{n^r}K_r^*.$$

Remarquons dans ce but que

$$f(x) - a_0 = \frac{1}{\pi}\int_0^{2\pi}\varphi_r(t)\,f^{(r)}(x+t)\,dt,$$

$$f^*(x) = \frac{1}{\pi}\int_0^{2\pi}\psi_r(t)\,f^{(r)}(x+t)\,dt,$$

où

$$\varphi_r(t) = \sum_{m=1}^{\infty}\frac{\cos\left(mt + \frac{\pi}{2}r\right)}{m^r}, \quad \psi_r(t) = \sum_{m=1}^{\infty}\frac{\sin\left(mt + \frac{\pi}{2}r\right)}{m^r}.$$

Par conséquent,

$$\left|f(x) - a_0 - \sum_{k=1}^{n-1}k^r\,\xi_k\,(a_k\cos kx + b_k\sin kx)\right| =$$

$$= \left|\frac{1}{\pi}\int_0^{2\pi}\left\{\varphi_r(t) - \sum_{k=0}^{n-1}\xi_k\cos\left(kt + \frac{\pi}{2}r\right)\right\}f^{(r)}(x+t)\,dt\right|$$

et

$$\left|f^*(x) - \sum_{k=1}^{n-1}k^r\,\eta_k\,(b_k\cos kx - a_k\sin kx\right| =$$

$$= \left|\frac{1}{\pi}\int_0^{2\pi}\left\{\psi_r(t) - \sum_{k=0}^{n-1}\eta_k\sin\left(kt + \frac{\pi}{2}r\right)\right\}f^{(r)}(x+t)dt\right|,$$

d'où

$$E_{n-1}(f) \leqslant \min \frac{1}{\pi} \int_0^{2\pi} \left| \varphi_r(t) - \sum_{k=0}^{n-1} \xi_k \cos\left(kt + \frac{\pi}{2} r\right) \right| dt, \tag{3}$$

$$E_{n-1}(f^*) \leqslant \min \frac{1}{\pi} \int_0^{2\pi} \left| \psi_r(t) - \sum_{k=0}^{n-1} \eta_k \sin\left(kt + \frac{\pi}{2} r\right) \right| dt. \tag{4}$$

Mais, comme il a été montré tout récemment par J. Favard (1), la seconde partie de (3) est égale à $\frac{4}{\pi} \frac{1}{n^r} K_r$ et ce minimum est atteint lorsqu'on détermine les nombres $\xi_k = \xi_k^\circ$ de telle manière que l'expression

$$\varphi_r(t) - \sum_{k=0}^{n-1} \xi_k \cos\left(kt + \frac{\pi}{2} r\right)$$

s'annule en même temps que la fonction $\cos\left(nt + \frac{\pi}{2} t\right)$; la relation (1) est donc établie.

Pour démontrer la relation (2) montrons d'abord que quels que soient les nombres η_k (que nous pouvons considérer comme réels), la différence

$$\psi_r(t) - \sum_{k=0}^{n-1} \eta_k \sin\left(kt + \frac{\pi}{2} r\right) \tag{5}$$

a $2n - \varepsilon$ zéros au plus dans l'intervalle $0 < t < 2\pi$. En effet, on a

$$\psi_1(t) = -\log\left(2 \sin \frac{t}{2}\right); \qquad (0 < t < 2\pi)$$

donc si pour $r = 1$ l'expression (5) a $2n + 1$ zéros dans l'intervalle $0 < t < 2\pi$, la différence

$$\frac{1}{2} \operatorname{ctg} \frac{t}{2} - \sum_{k=1}^{n-1} k\eta_k \sin kt,$$

et par suite le polynôme trigonométrique

$$\frac{1}{2} \cos \frac{t}{2} - \sum_{k=1}^{n-1} k\eta_k \sin kt \sin \frac{t}{2}$$

a $2n$ zéros au moins dans l'intervalle $0 < t < 2\pi$, ce qui est absurde.

Notre affirmation est ainsi justifiée pour $r = 1$. Pour $r > 1$ on y arrive en partant du théorème de Rolle et en remarquant que

$$\frac{d}{dt} \psi_r(t) = -\psi_{r-1}(t).$$

Donc, les nombres $\eta_k = \eta_k^\circ$ étant choisis de manière que l'expression (5) s'annule en même temps que la fonction $\sin\left(nt + \frac{\pi}{2} r\right)$, cette expression changera aussi de signe en même temps que $\sin\left(nt + \frac{\pi}{2} r\right)$. Pour ce choix des coefficients η_k° on a

$$\frac{1}{\pi} \int_0^{2\pi} \left| \psi_r(t) - \sum_{k=0}^{n-1} \eta_k^\circ \sin\left(kt + \frac{\pi}{2} r\right) \right| dt =$$

$$= \frac{1}{\pi}\int_0^{2\pi} \left\{\psi_r(t) - \sum_{k=0}^{n-1} \eta_k^0 \sin\left(kt + \frac{\pi}{2}r\right)\right\} \operatorname{sgn}\left\{\sin\left(nt + \frac{\pi}{2}r\right)\right\} dt =$$

$$= \frac{1}{\pi}\int_0^{2\pi} \psi_r(t) \operatorname{sgn}\left\{\sin\left(nt + \frac{\pi}{2}r\right)\right\} dt,$$

tandis que pour η_k quelconques

$$\frac{1}{\pi}\int_0^{2\pi} \left|\psi_r(t) - \sum_{k=0}^{n-1} \eta_k \sin\left(kt + \frac{\pi}{2}r\right)\right| dt \geqslant$$

$$\geqslant \frac{1}{\pi}\int_0^{2\pi} \left\{\psi_r(t) - \sum_{k=0}^{n-1} \eta_k \sin\left(kt + \frac{\pi}{2}r\right)\right\} \operatorname{sgn}\left\{\sin\left(nt + \frac{\pi}{2}r\right)\right\} dt =$$

$$= \frac{1}{\pi}\int_0^{2\pi} \psi_r(t) \operatorname{sgn}\left\{\sin\left(nt + \frac{\pi}{2}r\right)\right\} dt = \frac{4}{\pi n^r} K_r.$$

La relation (2) est donc aussi établie. En passant nous avons trouvé les inégalités suivantes:

$$\left|f(x) - a_0 - \sum_{k=1}^{n-1} k^r \xi_k^0 (a_k \cos kx + b_k \sin kx)\right| \leqslant \frac{4}{\pi n^r} K_r, \qquad (6)$$

$(f \subset H_r),$

$$\left|f^*(x) - \sum_{k=1}^{n-1} k^r \eta_r^0 (b_k \cos kx - a_k \sin kx)\right| \leqslant \frac{4}{\pi n^r} \leqslant K_r^*; \qquad (7)$$

qui, d'après (1) et (2), ne peuvent être améliorées.

L'inégalité (6) est une généralisation du théorème de J. Favard [1] ($a_0 = a_1 = \ldots = a_{n-1} = b_1 = \ldots = b_{n-1} = 0$) qui nous a permis d'obtenir nos résultats.

L'inégalité (7) permet de compléter le théorème de J. Favard en démontrant la proposition suivante:

Si la fonction

$$f(x) = \sum_{k=n}^{\infty} (a_k \cos kx + b_k \sin kx)$$

appartient à H_r, c'est-à-dire $|f^{(r)}(x)| \leqslant 1$, on a l'inégalité

$$\left|f^*(x)\right| = \left|\sum_{k=n}^{\infty} (b_k \cos kx - a_k \sin kx)\right| \leqslant \frac{4}{\pi n^r} K_r^*;$$

cette inégalité ne peut être améliorée.

Indiquons pour terminer les valeurs des nombres ξ_k^0, dont l'évaluation ne présente de difficulté,

$$k^r \xi_k^0 = -\frac{1}{(r-1)!} z^r \frac{d^{r-1}}{dz^{r-1}}\left(\frac{\pi}{\sin \pi z}\right) = D_r\left(\frac{k}{2n}\right)$$

pour r pair et

$$k^r \xi_k^0 = \frac{1}{(r-1)!} z^r \frac{d^{r-1}}{dz^{r-1}} (\pi \operatorname{ctg} \pi z) = D_r\left(\frac{k}{2n}\right)$$

pour r impair.

En introduisant ainsi les sommes généralisées

$$S_{n-1}(f; r) = \sum_{k=0}^{n-1} D_r\left(\frac{k}{2n}\right)(a_k \cos kx + b_k \sin kx)$$

on parvient à un procédé nouveau de sommation des séries de Fourier possédant cette propriété que pour toute $f \subset H_r$

$$\max_{0 \leqslant x \leqslant 2\pi} |f(x) - S_{n-1}(f; r)| \leqslant \sup_{g \subset H_r} E_{n-1}(g) = \frac{4}{\pi n^r} K_r,$$

donc, à un procédé de sommation que l'on doit considérer comme le meilleur, si l'on ne sait rien sur la fonction $f(x)$ sauf que $f \subset H_r$.

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LITTÉRATURE CITÉE

[1] J. Favard, Matematisk Tidsskrift, B, 81—94 (1936).

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MATHÉMATIQUES

SUR LA THÉORIE DES ENSEMBLES QUASI-MINIMAUX

Par H. HILMY

(Présenté par S. N. Bernstein de l'Académie, le 17. III. 1937)

1. Cette note est au fond un complément à mon mémoire «Sur les ensembles quasi-minimaux dans les systèmes dynamiques» (1). Cependant elle est écrite de manière à être intelligible aux lecteurs qui ne connaissent pas l'article cité. Notre intention est de montrer que si nous laissons de côté les rapports des ensembles quasi-minimaux avec d'autres types d'ensembles appartenant à la théorie générale des systèmes dynamiques, et surtout avec le type des systèmes dynamiques indécomposables (étudiés dans le mémoire cité) et si nous nous bornons à un problème plus restreint, savoir à l'examen des mouvements contenus dans les ensembles quasi-minimaux, notre but peut être atteint d'une façon plus simple et plus rapide. Il suffit pour cela d'adopter comme point de départ une définition de l'ensemble quasi-minimal qui, formellement, diffère de la définition donnée précédemment, mais qui, logiquement, lui est équivalente.

2. Considérons les mouvements des points dans l'espace métrique M; désignons par $f(p, t)$ la position du point $p \subset M$ à l'instant du temps t; $f(p, 0) = p$. Supposons que ces mouvements forment un groupe à un paramètre, c'est-à-dire que $f(f(p, t), \tau) = f(p, t + \tau)$. Nommons trajectoire l'ensemble de tous les points de la forme $f(p, t)$ avec p fixe et pour toute valeur réelle de t, et le point p — point initial de cette trajectoire. Nommons semi-trajectoire positive (négative) l'ensemble de tous les points de la forme $f(p, t)$ avec p fixe et pour toute valeur réelle de $t \geqslant 0$ ($t \leqslant 0$), et le point p — point initial de cette semi-trajectoire.

Supposons ensuite que par tout point $p \subset M$ passe une seule trajectoire.

Si A est un sous-ensemble arbitraire de M, nous désignerons par le symbole $f(A, t)$ l'ensemble $\{f(p, t)\}$, $p \subset A$.

Soient x et y deux points de l'ensemble M, le symbole $\rho(x, y)$ désignera alors la distance entre les points x et y.

Soit $x \subset M$ un point fixe, nous désignerons alors par le symbole $S(x, \varepsilon)$ l'ensemble de tous les points $y \subset M$ satisfaisant à la condition $\rho(x, y) < \varepsilon$.

Nommons cet ensemble voisinage sphérique du rayon ε du point x. Supposons que dans notre cas a lieu la dépendance continue des trajec-

toires des conditions initiales, habituelle pour les systèmes dynamiques, c'est-à-dire que pour tout complexe de nombres positifs arbitraires ε et T indépendants l'un de l'autre on peut indiquer un nombre positif δ assez petit pour que l'inclusion

$$f(S(p, \delta), t) \subset S(f(p, t), \varepsilon)$$

soit vérifiée pour toute valeur de t satisfaisant l'inégalité $|t| \leqslant T$.

Nous disons que l'ensemble $A \subset M$ est positivement (négativement) invariant si la condition $p \subset A$ implique $f(p, t) \subset A$ pour tout $t \geqslant 0$ ($t \leqslant 0$). Nous disons que l'ensemble A est invariant si c'est un ensemble à la fois positivement et négativement invariant.

Supposons que pour tout ensemble $A \subset M$ il existe une base dénombrable de domaines relatifs, c'est-à-dire une suite dénombrable d'ensembles relatifs ouverts $U_1, U_2, \ldots, U_n, \ldots$ telle que pour tout point $p \subset A$ et pour tout son voisinage sphérique S il se trouve un domaine U_n, qui satisfait à la condition $p \subset U_n \subset S$.

Nous allons considérer les mouvements stables au sens de Poisson. Le mouvement est dit stable au sens de Poisson si pour tout point p de sa trajectoire et pour tout couple de nombres positifs arbitraires ε et T il existe des valeurs de t, tant positives que négatives et supérieures par la valeur absolue à T, telles que $\rho(p, f(p, t)) < \varepsilon$.

3. Dans la suite nous aurons recours aux lemmes suivants que nous ne démontrons pas, vu leur évidence; notons seulement que le second lemme découle de la dépendance continue des trajectoires des conditions initiales.

Lemme I. *Soit A un ensemble invariant et $B \subset A$ un ensemble positivement (négativement) invariant, alors $A - B$ est un ensemble négativement (positivement) invariant.*

Lemme II. *La fermeture $\bar{A}$ de l'ensemble positif (négatif) invariant A est un ensemble positivement (négativement) invariant.*

4. Nous disons que l'ensemble invariant fermé Ω (qui peut ne pas être borné) est quasi-minimal s'il ne peut être représenté comme une somme de deux ensembles fermés, différents de Ω, dont l'un est positivement et l'autre négativement invariant.

Théorème I. *Soit Ω un ensemble quasi-minimal et Δ un sous-ensemble arbitraire positivement (négativement) invariant, alors l'ensemble Δ est dense partout dans Ω, ou bien il n'y est dense nulle part.*

Nous nous bornerons à démontrer le théorème en supposant que l'ensemble Δ est positivement invariant, car dans le cas où l'ensemble est négativement invariant la démonstration est toute analogue.

Supposons que l'ensemble Δ ne soit dense partout dans Ω et supposons, contrairement à l'hypothèse, qu'il n'y serait pas nulle part dense. Il se trouvera alors dans l'ensemble Ω un domaine relatif G dans lequel l'ensemble de points appartenant à Δ est dense partout et également un domaine G' dont aucun point n'appartient à Δ. Considérons l'ensemble positivement invariant de tous les points de la forme $f(p, t)$, où $p \subset \Delta \cdot G$ et $t \geqslant 0$. Désignons par Ω_1 la fermeture de cet ensemble; en vertu du lemme II l'ensemble Ω_1 est un ensemble positivement invariant. Il est évident que $G \subset \Omega_1$ et $G' \cdot \Omega_1 = 0$, et par suite $\Omega_1 \neq \Omega$.

Considérons l'ensemble $\Omega - \Omega_1$ et désignons par Ω_2 la fermeture de cet ensemble; en vertu du lemme I et du lemme II l'ensemble Ω_2 est un ensemble négativement invariant. Il est évident que $G' \subset \Omega_2$ et $G \cdot \Omega_2 = 0$, et par suite $\Omega_2 \neq \Omega$.

Il découle du mode de construction des ensembles Ω_1 et Ω_2 que

$$\Omega = \Omega_1 + \Omega_2,$$

Ω_1 étant un ensemble fermé positivement invariant et Ω_2 un ensemble fermé négativement invariant. Mais ceci contredit la supposition que l'ensemble Ω est un ensemble quasi-minimal et le théorème est démontré.

5. Démontrons à présent les théorèmes qui établissent le rapport des mouvements stables au sens de Poisson avec les ensembles quasi-minimaux.

Théorème II. *Tout mouvement stable au sens de Poisson* $f(p, t)$ *est contenu dans un ensemble quasi-minimal* Ω, *la trajectoire de ce mouvement étant partout dense dans* Ω.

Désignons par Ω l'ensemble des points appartenant à la fermeture de la trajectoire $f(p, t)$. L'ensemble Ω est invariant et fermé; de plus la trajectoire du mouvement est partout dense dans Ω. Montrons que l'ensemble Ω est un ensemble quasi-minimal.

Admettons qu'il n'en soit pas ainsi. Alors on pourrait indiquer deux ensembles fermés Ω_1 et Ω_2 différents de Ω, tels que $\Omega = \Omega_1 + \Omega_2$, et dont le premier est positivement et le second négativement invariant. Le point p appartient au moins à l'un des ensembles Ω_1 et Ω_2; admettons pour fixer les idées que $p \not\subset \Omega_1$. Vu nos suppositions il se trouvera un domaine relatif $G \subset \Omega - \Omega_1$. L'ensemble Ω étant un ensemble positivement invariant et $p \subset \Omega_1$, $f(p, t) \not\subset G$ pour $t \geqslant 0$, or ceci est en contradiction avec les propriétés du mouvement $f(p, t)$, qui est un mouvement stable au sens de Poisson avec la trajectoire partout dense dans Ω. Le théorème est démontré.

Théorème III. *Tous les points de l'ensemble quasi-minimal* Ω, *sauf un ensemble des points de la première catégorie de Baire, appartiennent aux trajectoires des mouvements stables au sens de Poisson, dont chacune est partout dense dans* Ω.

Désignons par Q l'ensemble des points appartenant à Ω et situés sur des trajectoires dont au moins une semi-trajectoire n'est pas partout dense dans Ω, et par Q^1 et Q^2—les ensembles des points situés sur des trajectoires dont les semi-trajectoires respectivement positives et négatives $Q = Q^1 + Q^2$ ne sont pas partout denses dans Ω.

Considérons les domaines $U_1, U_2, \ldots, U_n, \ldots$ formant une base dénombrable de l'ensemble Ω. Pour tout point $p \subset Q^1$ on peut indiquer un domaine U_ν tel qu'aucun point $f(p, t)$ pour $t \geqslant 0$ n'appartiendra au domaine U_ν. Désignons par $Q_1^1, Q_2^1, \ldots, Q_n^1, \ldots$ les ensembles de points qui sont les points initiaux des semi-trajectoires positives qui n'auront respectivement pas de parties communes avec les domaines $U_1, U_2, \ldots$ $\ldots, U_n, \ldots$ Chacun des ensembles Q_i^1 est un ensemble positivement invariant et ne peut être partout dense dans Ω; mais alors en vertu du théorème I chacun des ensembles Q_i^1 n'est dense nulle part dans Ω.

D'autre part il est évident que

$$Q^1 = \sum_{i=1}^{\infty} Q_i^1.$$

Ainsi l'ensemble Q^1 est la somme dénombrable des ensembles Q_i^1 dont aucun n'est dense nulle part dans Ω, c'est-à-dire qu'il est un ensemble de la première catégorie de Baire. D'une manière analogue on peut dé-

montrer que l'ensemble Q^2 est un ensemble de la première catégorie de Baire. Mais alors l'ensemble $Q = Q^1 + Q^2$ doit également être un ensemble de la première catégorie de Baire, relativement à Ω.

L'ensemble $P = \Omega - Q$, étant le complément d'un ensemble de la première catégorie, ne peut être vide et toute trajectoire contenue dans P est la trajectoire d'un mouvement stable au sens de Poisson, car ses deux semi-trajectoires sont partout denses dans Ω. Le théorème est démontré.

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LITTÉRATURE CITÉE

[1] H. Hilmy, Annals of Mathematics, 37 (1936).

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MATHÉMATIQUES

SUR LA DISTRIBUTION DES VALEURS DE QUELQUES FONCTIONS ARITHMÉTIQUES

Par R. O. KUZMIN

(Présenté par S. N. Bernstein, de l'Académie, le 23. III. 1937)

M. G. Hardy et S. Ramanujan ont démontré que pour la plupart des nombres n on a l'inégalité $d(n) < (\log n)^{0.7}$, où $d(n)$ est le nombre des diviseurs de n. Cela veut dire que pour $x \to \infty$ on a $\frac{\nu(x)}{x} \to 1$, où $\nu(x)$ est le nombre des valeurs de n dans l'intervalle $(1, x)$ pour lesquels $d(n) < (\log n)^{0.7}$. On peut dire que l'ordre normal de grandeur de $d(n)$ est au plus $(\log n)^{0.7}$. Mais Gauss et Dirichlet ont demontré qu'on a:

$$\sum_{n \leqslant x} d(n) = x \log x + O(x).$$

Donc on peut conclure que la valeur moyenne des nombres $d(n)$ est d'ordre $\log n$. En comparant les résultats mentionnés on peut dire que parmi les nombres $d(n)$ il y en a qui ont une grandeur exceptionnelle dont l'ordre est plus grand que l'ordre normal des nombres $d(n)$. La même conclusion résulte d'une formule, due à M. K. Mardjanichwili, d'après laquelle on a:

$$\sum_{n \leqslant x} d^2(n) = x(A_1 \log^3 x + A_2 \log^2 x + A_3 \log x + A_4) + O(x^{\frac{7}{8}} \log x).$$

Par une méthode tout à fait différente j'ai réussi d'obtenir une formule générale, qui est liée avec l'existence des valeurs exceptionnellement grandes de quelques fonctions arithmétiques. Le théorème fondamental est le suivant.

Soit $f(s)$ une fonction qui est, pour Re $s > 1$, présentée dans les deux formes:

$$f(s) = \prod_p \left(1 + \sum_{m=1}^{\infty} \frac{b_m}{p^{ms}}\right) = \sum_{n=1}^{\infty} \frac{c_n}{n^s}.$$

Les nombres b_m sont réels, $b_m = O(m^k)$, où k est un nombre donné. Supposons encore que $b_1 = b$ est un entier positif.

Dans ces conditions, ν étant un entier positif quelconque, on a la formule:

$$\sum_{n\leqslant x} a_n^{\nu} = x \sum_{\mu=1}^{b^{\nu}} A_{\mu} \log^{b^{\nu}-\mu} x + O\left(x^{\frac{b^{\nu}+3}{b^{\nu}+6}}\right). \qquad (*)$$

On peut encore préciser le reste dans le second membre. Si l'hypothèse célèbre de Riemann est vraie, le reste a la forme $O\left(x^{\frac{1}{2}+\varepsilon}\right)$, ε étant un nombre positif arbitraire.

La démonstration est fondée sur une simple remarque suivant laquelle la fonction $f(s)$ peut être représentable dans le demi-plan

$$\operatorname{Re} s > \frac{1}{2} + \delta \qquad \delta > 0$$

sous la forme: $f(s) = [\zeta(s)]^b \cdot F(s)$. La fonction $F(s)$ est la somme d'une série de Dirichlet dont la somme est bornée: $|F(s)| < M = M(\delta)$. Il faut tenir compte des formules: $a_n = O(n^{\varepsilon})$ et $\zeta\left(\frac{1}{2}+ti\right) = O(t^{\rho})$; $\rho < \frac{1}{6}$. En partant de ces remarques on peut étudier la somme $\sum d(n)$ par la méthode générale de M. E. Landau pour l'évaluation des sommes des coefficients d'une série de Dirichlet.

Le cas plus général d'une somme $\sum a_n^{\nu}$ peut être réduit au cas particulier, où $\nu = 1$.

En comparant les formules (*) pour les mêmes a_n et des valeurs differentes de ν ($\nu = 1, 2, 3, \ldots$) on peut obtenir quelques résultats sur la distribution des valeurs de a_n.

Indiquons encore deux cas particuliers intéressants de la formule (*).

1. $b_m = m + 1$; $a_n = d(n)$ est le nombre des diviseurs de n:

$$\sum_{n\leqslant x} d^{\nu}(n) = x \sum_{\mu=1}^{2^{\nu}} A_{\mu} \log^{2^{\nu}-\mu} x + O\left(x^{\frac{2^{\nu}+3}{2^{\nu}+6}}\right).$$

Si $\nu = 2$ on a:

$$\sum_{n\leqslant x} d^2(n) = x \sum_{\mu=1}^{\mu=4} A_{\mu} \log^{4-\mu} x + O\left(x^{\frac{7}{10}}\right).$$

C'est la formule de M. K. Mardjanichwili, mais la forme du reste est un peu différente.

2. $b_m = 2$; $a_n = D(n)$ est le nombre des diviseurs de n qui n'ont aucun carré pour diviseur. Dans ce cas on a la formule:

$$\sum_{n\leqslant x} D^{\nu}(n) = x \sum_{\mu=1}^{2^{\nu}} B_{\mu} \log^{2^{\nu}-\mu} x + O\left(x^{\frac{2^{\nu}+3}{2^{\nu}+6}}\right).$$

Ces formules montrent que dans les sommes $\sum d^{\nu}(n)$ et $\sum D^{\nu}(n)$ l'influence des termes de grandeur exceptionnelle croît avec ν.

Je donnerai dans un autre receuil un exposé détaillé et quelques conséquences de ces formules.

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LITTÉRATURE CITÉE

[1] Strinivasa Ramanujan, Collected Papers, Cambridge (1927).
[2] К. К. Марджанишвили, Изв. Тифлисского ун-та (1929).
[3] E. Landau, Handbuch der Lehre von der Verteilung der Primzahlen, Berlin-Leipzig. Bd. II, S. 238 (1909).

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ASTRONOMIE

LES ECLIPSES DE LA LUNE ET LA DISTRIBUTION DE L'OZONE ATMOSPHÉRIQUE

Par B. FESSENKOFF, de l'Académie

La luminosité à l'intérieur de l'ombre terrestre projeté sur la Lune dépend du pouvoir réfrangeant de l'atmosphère terrestre agissant comme une lentille convergente, du gradient de la réfraction suivant la hauteur, de l'absorption de lumière passant à travers l'atmosphère et enfin de la diffusion de la lumière du Soleil par toutes les couches atmosphériques.

Le dernier facteur peut certainement être negligé pour les régions de l'ombre au voisinage de son bord. Ces régions sont d'ailleurs les seules intéressantes au point de vue de la constitution de la stratosphère. Considérons d'abord un seul élément da la surface du Soleil. La luminosité de l'ombre à la distance ρ du centre peut alors être représentée par le produit:

$$f(\rho) = \mathrm{I} \cdot \mathrm{II} \cdot \mathrm{III} \frac{dy}{dh_0} = \left(1 - \frac{2lR}{r_0 + h_0}\right)^{-1} \left(1 - 2l \frac{dR}{dh_0}\right)^{-1} e^{-f(h_0)} \frac{dy}{dh_0},$$

où R est la réfraction horizontale qui charactérise la trajectoire lumineuse passant à la hauteur minimale h_0, $f(h_0)$ — la masse optique de l'atmosphère comptée suivant la trajectoire correspondante, r_0 — le semi-diamètre de la Terre, l — la distance de la Lune. Le coefficient supplémentaire $\frac{dy}{dh_0}$ est introduit pour tenir compte de la divergeance des rayons du faisceau primitivement homogène qui s'accuse le long de la trajectoire à partir de l'entrée dans l'atmosphère jusqu'au point le plus proche de la Terre. Sa valeur généralement est peu différente de l'unité.

Pour le calcul de la réfraction horizontale on peut se servir de la série procédant suivant les puissances de l'angle de l'amplitude φ compté au centre de la Terre. La forme générale de cette série est

$$R = \varkappa_1 \varphi + (\varkappa_1 + \varkappa_1^2 + \varkappa_2) \operatorname{ctg} \zeta \frac{\varphi^2}{2} + [(1 - \varkappa_1)(\varkappa_1 + \varkappa_1^2 + \varkappa_2) +$$

$$+ (2\varkappa_1 + 3\varkappa_1^2 + 4\varkappa_2 + \varkappa_1^3 + 2\varkappa_1\varkappa_2 + \varkappa_3) \operatorname{ctg}^3 \zeta] \frac{\varphi^3}{6} + \ldots,$$

où

$$\varkappa_1 = -\frac{r}{n}\frac{dn}{dr}; \quad \varkappa_2 = -\frac{r^2}{n}\frac{d^2n}{dr^2} \text{ etc}; \quad n = 1 + c\delta; \quad \frac{d\delta}{\delta} = -\frac{dh}{\tau}\left(\frac{g}{\gamma} + \frac{d\tau}{dh}\right)$$

δ étant la densité de l'air à la hauteur $h = r - r_0$, g — l'accélération de pesanteur, τ — température absolue à la même hauteur, ζ — l'angle entre la tangente à la trajecteire et le rayon-vecteur. En appliquant ensuite la théorie généralisée de l'absorption atmosphérique de Laplace et la rattachant aux observations appropriées produites aux sommets des montagnes, on obtient les valeurs numériques suivantes de la fonction $f(\rho)$:

Table 1

h_0 (km)	$\frac{\rho}{r_0}$	I	II	III	$\frac{dh_0}{dy}$	$f(\rho)$
8	0.461	2.17	0.00280	0.0106	1.100	$0.579 \cdot 10^{-4}$
12	0.654	1.53	0.00344	0.0831	1.070	$0.406 \cdot 10^{-3}$
14	0.745	1.342	0.00357	0.168	1.056	$0.756 \cdot 10^{-3}$
16	0.825	1.212	0.00455	0.291	1.045	$0.533 \cdot 10^{-3}$
18	0.877	1.140	0.00654	0.420	1.031	$3.03 \cdot 10^{-3}$
21	0.933	1.072	0.01168	0.621	1.015	$7.65 \cdot 10^{-3}$
26	0.972	1.029	0.0347	0.817	1.004	0.0290
31	0.986	1.014	0.091	0.912	1.002	0.0840
36	0.993	1.007	0.13	0.946	1.001	0.124
41	0.996	1.0035	0.18	0.981	1.000	0.177

On voit que la luminosité de l'ombre près de sa limite dépend principalement du gradient de la réfraction $\frac{dR}{dh_0}$.

Passons maintenant à la discussion de l'influence de l'ozone atmosphérique. Supposons pour fixer les idées que la même masse de l'ozone forme une couche homogène entre les limites des hauteurs a) 20—21 km, b) 21—26 km, c) 21—31 km. L'examen de la transmission atmosphérique pour les λ différents fournit la possibilité d'obtenir la valeur du coefficient d'absorption pour la bande de Chapuis. J'en ai obtenu 0.069, ce qui correspond à la quantité moyenne de l'ozone. On peut alors calculer la luminosité à l'intérieur de l'ombre dans les trois cas considérés, ce qui est donné dans la table suivante:

Table II

$\frac{\rho}{r_0}$	$f(\rho)$ sans ozone	$f(\rho)$ avec la couche d'ozone de		
		1 km	5 km	10 km
1.00	100	100	100	100
0.98	5.20	5.20	5.20	1.14
0.96	1.75	1.75	0.174	0.232
0.94	0.93	0.93	0.0391	0.0890
0.92	0.59	0.001	0.0603	0.0976
0.90	0.42	0.0201	0.0697	0.0933
0.88	0.318	0.0295	0.0620	0.0794
0.86	0.234	0.0307	0.0519	0.0632
0.84	0.179	0.0284	0.0438	0.0516
0.82	0.145	0.0261	0.0381	0.0438
0.80	0.119	0.0235	0.0334	0.0377
0.70	0.0540	0.0142	0.0178	0.0198
0.60	0.0290	0.0086	0.0107	0.0109
0.50	0.0086	0.0027	0.0031	0.0033
0.40	0.0046	0.0014	0.0017	0.0018

On doit maintenant chercher l'influence sommaire de tous les éléments du disque solaire en calculant l'expression

$$f(\bar{\rho}) = \int\int f(\rho)\,\varphi(\varepsilon)\,d\sigma,$$

où $\varphi(\varepsilon)$ représente la distribution de l'intensité lumineuse sur le disque du Soleil. La luminosité dans l'ombre obtenue de cette façon par comparaison avec celle hors de l'eclipse est donnée par la table suivante:

Table III

$\frac{\rho}{r_0}$	Sans ozone	Couche homogène de l'ozone		
		1 km	5 km	10 km
0.60	0.000424	0.000103	0.000137	0.000152
0.66	0.000829	0.000135	0.000203	0.000241
0.69	—	0.000384	0.000243	0.000300
0.72	0.00243	0.00210	0.00176	0.00158

Les observations photométriques de Keenan (eclipse 27.XI.1928) et celles de Danjon (eclipse 26.IX.1931) comparées avec les résultats précedents conduisent à penser que l'ozone se propage bien au dessus de sa limite inférieure de 20 km et à occupe une couche peu près de 30 km en largeur. Les observations photométriques ultérieures en lumière rouge et verte sont hautement désirables pour élucider définitivement ce point.

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ASTRONOMIE

L'UNIVERS INFINI ET LA LUMINOSITÉ DU CIEL NOCTURNE

Par B. FESSENKOFF, de l'Académie

La luminosité du ciel nocturne est un phénomène fort compliqué qui dépend au moins de 12 causes différentes. Les voici: 1) émission monochromatique des basses couches stratosphériques; 2) émission monochromatique des couches stratosphériques élevées; 3) crépuscule électronique; 4) prolongement électronique de la couronne solaire; 5) courants d'électrons dans le champ magnétique de la Terre; 6) accumulation de la matière météorique près du plan invariable du système solaire; 7) milieu diffusant commun à l'espace interstellaire assujetti au rayonnement du Soleil; 8) matière liée avec l'anneau des astéroïdes; 9) matière condensée près du point de libration sur la droite Soleil — Terre au delà de l'orbite de la Lune; 10) éclat intégral de toutes les étoiles de notre système galactique; 11) diffusion de la lumière des étoiles par le milieu interstellaire; 12) éclat intégral de toutes les galaxies qui forment un système du second ordre.

Les trois premières causes liées avec l'atmosphère sont les plus importantes. J'ai montré ailleurs comment on-peut par des observations spéciales éliminer la composante terrestre, quel que soit sa nature. On obtient alors par exemple pour le pôle céleste la luminosité dépendant uniquement des causes cosmiques égale à peu près à 0.7—0.8 en nombre d'étoiles de la cinquième grandeur par degré carré. En se basant sur les données numériques de la statistique stellaire on peut évaluer l'éclat intégral des étoiles à 0.4. En outre, il faut croire que les causes planétaires doivent contribuer non moins que 0.1—0.2 pour le même point du ciel. Par conséquent, il ne paraît pas possible d'attribuer aux causes 11) et 12) une valeur supérieure à 0.2.

Il est connu à présent qu'il existe dans le plan galactique une couche assez mince de la matière absorbante et diffusante. Les différents auteurs estiment que l'absorption dans cette couche est égale à 1 magnitude stellaire par un kiloparsec. Nous pouvons donc sans nous éloigner beaucoup de la réalité considérer un schème suivant: une couche de largeur H, limitée par deux plans parallèles et infinis est remplie uniformément par les étoiles et la matière diffusante. Nous supposons essentiellement que l'affaiblissement de lumière dans ce millieu se produit à cause de la diffusion indépendante de la direction. Par conséquent, le coefficient de la diffusion μ et de l'absorption k sont liées par une expression simple

$$k = 4\pi\mu.$$

La quantité de lumière émise par l'unité de volume peut être représentée par une série

$$J = nL\Phi(y) = nL[1 + \Phi_1(y) + \Phi_2(y) + \Phi_3(y) + \ldots],$$

où les fonctions Φ_1, Φ_2, ... etc. caractérisent la diffusion de lumière des différents ordres et dépendent uniquement de la distance comptée à partir d'un plan limitant. On a:

$$\Phi_1(y) = 1 - \frac{1}{2}e^{-k(H-y)} - \frac{1}{2}e^{-ky} - \frac{1}{2}k(H-y)\,Ei[-k(H-y)] - \\ - \frac{1}{2}kyEi(-ky)$$

et généralement

$$\Phi_{i+1}(y) = -\frac{k}{2}\int_y^H \Phi_i(ky')\,Ei[-k(y'-y)]\,dy' - \\ - \frac{k}{2}\int_0^y \Phi_i(y')\,Ei[-k(y-y')]\,dy'.$$

Pour un observateur immérgé dans cette couche et placé à la distance h_0, la luminosité apparente du ciel, qui dépend uniquement de l'éclat intégral d'étoiles, doit être

$$J_* = \frac{nL}{k}[1 - e^{-k(H-h_0)\sec\zeta}],$$

tandis que celle qui dépend uniquement de la matière diffusante est

$$J = nL\int_{h_0}^H \Phi(ky)\,e^{-k(h-h_0)\sec\zeta}\sec\zeta\,dh,$$

ζ étant l'angle entre le rayon de la vision et la normale au plan limitant. Pour une couche dont la masse optique est égale à l'unité, les fonctions $\Phi_i(y)$ sont les suivantes:

ky	$\Phi_0(y)$	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5	$\Phi(y)$
0.0	1.000	0.426	0.252	0.159	0.106	0.075	1.120
0.5	1.000	0.674	0.448	0.291	0.187	0.125	1.930

Si, d'autre part, la masse optique est infinie, nous avons

$$\Phi_0 = \Phi_1 = \Phi_2 = \ldots = 1 \quad \text{et} \quad \Phi(y) = \infty,$$

c'est-à-dire la contribution de la part de la matière diffusant la lumière des étoiles est infiniment plus grande que des étoiles elles mêmes ce qui se comprend facilement comme effet de l'accumulation de l'énergie stellaire émise auparavant. Il est facile à voir que pour contribuer de 50% seulement à l'éclat direct d'étoiles, la matière diffusante ne doit avoir la masse optique plus grande que 0.3, ce qui est parfaitement admissible au point de vue d'observation. On peut admettre que le ciel nocturne doit paraître bien sombre pour un observateur placé loin au dela de la galaxie, peut être à 0.01 en nos unités au maximum.

Évaluons maintenant l'éclat intégral de toutes les galaxies qui sont accessibles à l'observation. En adoptant que la sphère de rayon R à 43 millions parsecs contient 2 millions galaxies (N) distribuées uniformé-

ment, et en se servant des résultats de Hubble sur la distribution de leur éclat absolu on obtient facilement

$$j = \frac{3}{4}\frac{N}{4\pi R^2}\left(\frac{\pi}{180}\right)^2 10^{-0{\cdot}4(\overline{M}-10)},$$

$\overline{M}$ étant l'éclat moyen d'une galaxie égale à —14.2.

On voit que la luminosité intégrale j croît proportionnellement au rayon de la métagalaxie, qui par conséquent doit être limitée. Son rayon ne peut probablement dépasser plus que 20 fois celui de l'observation.

En procédant de la même façon nous pouvons imaginer l'existence des systèmes d'ordres supérieurs jusqu'à l'infini. On peut montrer que la luminosité résultante est représentée dans ce cas par une série

$$J = \frac{3}{4}\frac{L}{\pi}\left(\frac{n_1}{R_1^2} + \frac{n_1 n_2}{R_2^2} + \frac{n_1 n_2 n_3}{R_3^2} + \ldots\right),$$

abstraction faite de la matière diffusante, R_i étant le rayon du système de i-ème ordre, et n_i le nombre qu'il contient de systèmes de $(i-1)$-ième ordre. La série doit être nécessairement convergente pour ne pas contredire les observations. La condition en est

$$\frac{n_i R_{i-1}^2}{R_i^2} \leqslant q < 1. \qquad (1)$$

Dans le cas actuel, de la matière diffusante appropriée à chaque système, la série est la suivante

$$J = \frac{3}{4}\frac{L}{\pi}\left(\frac{n_1}{R_1^2}\delta_1 + \frac{n_1 n_2}{R_2^2}\delta_2 + \frac{n_1 n_2 n_3}{R_3^2}\delta_3 + \ldots\right),$$

δ_i étant les fonctions assez compliquées ne dépendant que de la masse optique totale $k_i R_i$ du système correspondant. La série reste convergente à condition que

$$\delta_1 = \delta_2 = \delta_3 = \ldots \text{ const}$$

ou

$$k_1 R_1 = k_2 R_2 = k_3 R_3 = \ldots \qquad (2)$$

Par conséquent, pour bâtir l'univers infini sans entrer en contradiction avec les observations il faut se soumettre aux conditions (1), (2) quant aux dimensions des systèmes consécutives et à l'absorption dans leur milieu.

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PHYSICS

AN INVESTIGATION OF STRUCTURES OF SILICA GLASS, PUZZOLANS AND CLAYS BY MEANS OF ELECTRON DIFFRACTION

By N. A. ŠIŠAKOV (N. A. SHISHACOW)

(Communicated by V. A. Kistiakovskij, Member of the Academy, 7. III. 1937)

Both silica glass and that silicon dioxide which is the main constituent of the so-called puzzolans (tripolites, pumices, diatomites etc.) have so far been regarded as true amorphous substances. Several attempts were made to detect, if possible, the finest crystallites in silica glass by means of X-ray diffraction, but data obtained were always so scanty that the «crystallite theory» has not yet proved successful. In this respect even the X-ray diagrams with five diffuse reflections recently obtained by Valenkow and Poray-Koshiz (1) seem to be inadequate. As to the puzzolans, there are in literature, as far as I know, no indications of their crystalline state.

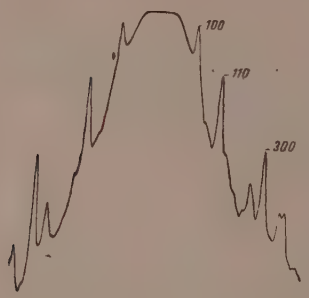
I have endeavoured to obtain more distinct diffraction patterns from these puzzling substances by means of electron rays in the hope that their shorter wave-length, as compared with that of X-rays, would be of assistance in detecting crystallites so minute as to be practically beyond the range of X-ray investigation. In fact, I have been successfull in obtaining electron diagrams with surprisingly thin prominent rings, first from vitreous silica (2) and then from some puzzolans (4). The intensity distribution on these diagrams appeared in both cases very similar to that of high-cristobalite, although the Bragg's spacings showed some discrepancies (3). For this reason it had to be assumed that the vitreous silica consists of slightly deformed high-cristobalite crystals. The same conclusion was arrived at with regard to pumice crystals (4).

Recent experiments have demonstrated that the above diffraction patterns with cristobalite distribution of intensities appears in the case of many other puzzolans also. It is hardly possible to explain the similarity of all these patterns by the influence of any contaminations as, for instance, of organic molecules. In fact, the diffraction patterns of vitreous silica and of puzzolans are quite different from those of organic substances, and, besides, as has already been pointed out in another paper, every silica modification gives its own characteristic electron diffraction pattern. Thus, ordinary sand gives an excellent pattern of low-quartz (4). Dinas brick with 80% of silica gives equally good specific diffraction pattern of low-cristobalite. Both show no rings, which are observed in the case of silica glass and puzzo-

lans. Similarly, no such «high-cristobalite» rings have been observed in the case of tridymite, silica gels, nor in the case of some opals.

As the above diffraction patterns were all obtained from powders remaining on a celluloid film after drying a drop of water suspension, one may assume that the patterns were due rather to silica hydroxide, than to anhydrous silica. It is believed, however, that this assumption is improbable, because a powder, which was dried at 110° and collected on a celluloid film by sedimentation in dry air, gave exactly the same pattern as that observed in the case of wet sedimentation.

Among the other substances investigated in the course of this work, very important are different clays. As may be seen from X-ray diagrams clay minerals always give a great number of weak reflections due to the low symmetry of their lattices. To my surprise not only did none of the electronograms show their characteristic reflections, but they all appear to be very similar to the electronograms of puzzolans and vitreous silica. It may not be out of place to note here that such patterns from clays are usually much more distinct than in the last case. A microphotogram of such a diffraction pattern is shown in the figure. To sum up, the above mentioned experiments have given almost quite identical patterns, which are especially distinct in the case of clays, but contain some background and a few «extra» rings in the case of silica glass and of some puzzolans.



In order to find the indices of the rings observed, it was decided to use only the electronograms without such extra rings, which are presumably not characteristic ones. As a more exact knowledge of Bragg's spacings was rather desirable, a repeated exposures of the best specimens of clays and puzzolans were made after mixing them with some quantity of pure sodium chloride as a standard substance, the parameter of that salt being assumed equal to 5.626 Å. Data relating to the most distinct and typical electronograms of clays and puzzolans are summarized in the table. The intensities I in the first column represent the average intensitites of different electronograms. Bragg's spacings d and the lattice constant a are expressed in Angström units. This constant was obtained on the assumption that the lattice in question was hexagonal. The assumption that the lattice is cubic or tetragonal or rhombohedric does not lead to satisfactory indices. As to the rhombic, monoclinic or triclinic syngonies, they are quite improbable, because the above patterns, since they contain only a few reflections, must, of course, belong to comparatively symmetrical lattices.

As may be seen from this table, the accuracy in determination of the constant a is as high as 0.5 per cent. Therefore, the indices selected ought to be correct. It will further be seen that they display the following regularities: first, that the third index is equal to zero for all reflections, and, second, that for the couples of (hk)-indices all possible combinations of integers have been used in succession. It is thus clear that the above characteristic patterns are exhaustive in the sense that they can contain no other reflections, except, perhaps (430). It is quite probable, however, that the latter is weaker than the (510) and (520) reflections, which, lie almost beyond the vision boundary. As to the condition $l=0$, it can

Typical Electronograms of Some Koalins and Clays

I	*d*	*a*	*(hkl)*
20	4.450	5.138	(100)
15	2.561	5.122	(110)
0.5	2.235	5.162	(200)
5	1.688	5,157	(210)
9	1.491	5.165	(300)
3	1.291	5.164	(220)
3	1.240	5.162	(310)
1	1.120	5.173	(400)
0.5	1.028	5.174	(320)
0.5	0.974	5.154	(410)
0.5	0.895	5.167	(500)
0.25	0.858	5.148	(330)
0.25	0.842	5.145	(420)
Not seen	—	—	(430)
Traces	0.808	5.195	(510)
»	0.720	5.192	(520)
		Average 5.161 Å	

only mean that the crystals in question have no third dimension.

It is well known that the diffraction of a monochromatic ray from a two-dimensional monocrystal should give some system of spots. But, in the case of diffraction of the same ray from a powder with a random orientation of finet wo-dimensional crystals, the diffraction pattern, according to Laue ([5]), must consist of a system of rings. Therefore, the rings observed may also be explained from this point of view.

It will be easy to conceive the structure of my two-dimensional crystals by ascribing the following coordinates to the initial points of Si- and O-lattices:

$$\text{Si} \quad (0,\ 0);\ \left(\frac{2}{3},\ \frac{2}{3}\right),$$

$$\text{O} \quad (0,\ 0);\ \left(\frac{2}{3},\ \frac{2}{3}\right);\ \left(\frac{2}{6},\ \frac{5}{6}\right);\ \left(\frac{5}{6},\ \frac{2}{6}\right);\ \left(\frac{5}{6},\ \frac{5}{6}\right).$$

These data are in agreement with the requirement of tetrahedral coordination of oxygen atoms around silicon atoms. The bonds between the tetraheders of SiO_4 are in this case the same as in silicates of the mica type, endless firm sheets of a Si_2O_5 being formed in either case. The same Si_2O_5 sheets, alternating, of course, with hydroxyl- and hydroalumina-sheets, also exist in every clay mineral.

If we pass over from the ordinary hexagonal unit cell with the above single parameter $a=5.161$ Å to the corresponding parameters a and b of the orthohexagonal unit cell, we shall have:

$$a = 5.161\,\text{Å} \quad \text{and} \quad b = 8.939\,\text{Å}$$

These data are in complete agreement with the known parameters a and b of some micas and clay minerals. We have, thus:

for muscovite	$a = 5.17$ Å:	$b = 8.94$ Å,
for koalinite	$a = 5.14$ Å;	$b = 8.90$ Å,
for metahalloysite	$a = 5.15$ Å;	$b = 8.90$ Å.

It is known, that all clay minerals are thermally unstable, their structures being broken up at 400—500° C. It is quite conceivable, therefore, that such a partial breaking-up of their lattices may also take place under the influence of the cathode ray. In such a way the fact may be explained that electronograms of clay minerals always show only reflection from the remaining firm Si_2O_5 sheets, instead of a great number of reflections from low-symmetrical unbroken clay minerals. As to the fact, that free alumina, which is always produced by such thermal decomposition, yields no diffraction pattern in spite of its comparatively high percentage, it seems reasonable to think that it is amorphous and that its influence on the whole diffraction pattern is as negligible, as that of a celluloid film.

The fact that diffraction patterns are almost exactly the same both in the case of clays and in the case of puzzolans and vitreous silica indicates that the latter also undoubtedly consist of two-dimensional crystals. It is believed that this similarity of structure is responsible for the fact that all these substances are to some extent similar in their properties as, for instance, in adsorption activity. The X-ray data show that none of the puzzolans contains clay minerals, so that the above two-dimensional crystals of Si_2O_5 are their main constitutent. Ordinary quartz, which is sometimes observed in puzzolans, as well as in some clays, is not an important constituent.

Now, the question arises as to the origin of the extra rings which are observed on the electronograms of some puzzolans and vitreous silica. For lack of more complete data it is at present difficult to draw any definite conclusion, but it is quite possible, that these extra rings may be due to the presence of a second crystalline phase or, rather, to aggregation of above two-dimensional crystals in the third dimension with the resulting formation of cristobalite-like structure. In support of the latter suggestion mention may be made of Niuwenkamp's work ([6]), who has shown, by means of X-ray analysis, that cristobalite and tridymite are built up of similar crystals of two-dimensional cristobalite and that the difference is only in that these crystals are either differently arranged relatively to one another. Whether or not this two-dimensional cristobalite is identical with the above two-dimensional crystals of puzzolans and silica glass, is now not yet cleared. up.

I hope to publish shortly a continuation of this work in the C. R. Acad. Sci. URSS and a full account of it in the Journal of Technical Physics (Russ.) and in Acta Physicochimica USSR.

The experimental part of this work was carried out in Cement Institute.

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REFERENCES

[1] N. Valenkowa. E. Poray-Koshiz, Nature, **137**, 273 (1936). [2] N. A. Shishacow, Nature, **136**, 514 (1935). [3] Н. А. Шишаков, Журн. техн. физ., **5**, 1834, (1935). [4] N. A. Shishacow, C. R. Acad. Sci. URSS, I(10), 15 (1936). [5] M. v. Laue, ZS. f. Krist., **82**, 127 (1932). [6] W. Niuwenkamp, ZS. f. Krist., **90**, 377 (1935).

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PHYSICS

ON THE SCATTERING OF LIGHT BY THE ELECTROSTATIC FIELD OF A POINT CHARGE ON THE NON-LINEAR QUANTUM ELECTRODYNAMICS

By S. ŠUBIN and A. SMIRNOV

(Communicated by L. I. Mandelstam, Member of the Academy, 20. III. 1937)

On the non-linear electrodynamics the presence of the electrostatic field of a point charge may disturb in a certain way the properties of the electromagnetic light wave in the neighbourhood of the charge. It is easy to see that this disturbance can be described partly as an appearance of scattered light. In the present article we shall discuss this scattering effect from the standpoint of the simplest form of the non-linear theory.

1. The energy density of the electromagnetic field (if it does not contain explicitly any derivatives of the field vectors) must in each theory be represented by the expression of the type

$$U = \frac{b^2}{4\pi} W\left(\frac{D^2}{b^2}, \frac{B^2}{b^2}, \frac{(DB)^2}{b^4}\right), \tag{1}$$

where D and B are the electric and magnetic field strengths * and b is the critical field.

We put

$$D = D_0 + D_{tr}, \quad B = B_{tr}, \tag{2}$$

where D_0 is the longitudinal electrostatic field of the point charge and D_{tr}, B_{tr} — the transverse fields of the light waves. As the longitudinal part of the field is not subject to quantization, we can simply write (with usual notations):

$$D_0 = Ze\frac{r}{|r|^3}. \tag{3}$$

Let us introduce (2) into (1) and develop the function W in powers of D_{tr} and B_{tr}, neglecting all the powers higher than the second (this means that we neglect all the a priori possible transitions in which more than two quanta take part). This gives

$$W = W^{(0)} + \frac{2D_0D_{tr} + D_{tr}^2}{b^2} W_1^{(0)} + \frac{B_{tr}^2}{b^2} W_2^{(0)} + \frac{(D_0B_{tr})^2}{b^4} W_3^{(0)} + 2\frac{(D_0D_{tr})^2}{b^4} W_{11}^{(0)}, \tag{4}$$

* For the sake of simplicity we do not use in this paper any special notations for the vector quantities.

where, for instance, $W_1^{(0)}$ denotes the value of the derivative of the function W with respect to the first argument for $D_{tr} = B_{tr} = 0$; an analogous meaning have also the other abbreviations.

Remembering that the unperturbed energy density is equal to

$$\frac{b^2}{4\pi} W^{(0)} + \frac{D_{tr}^2 + B_{tr}^2}{8\pi}$$

and using the usual expansions for D_{tr} and B_{tr}

$$D_{tr} = \sqrt{\frac{2\pi hc}{V}} \sum_k \sqrt{|k|}\, D_k\, e^{ikr}, \quad B_{tr} = \sqrt{\frac{2\pi hc}{V}} \sum_k \sqrt{|k|}\, B_k\, e^{ikr} \tag{5}$$

(h is the Planck's constant divided through 2π, V is the volume of the Hohlraum), we obtain the following expression for the perturbation energy of the interaction between the electrostatic field and the field of the light waves:

$$H' = \frac{hc}{2V} \sum_{k',\, k''} \sqrt{|k'|\,|k''|} \int \Big[D_{k'} D_{k''} \Big(W_1^{(0)} - \frac{1}{2} \Big) + B_{k'} B_{k''} \Big(W_2^{(0)} - \frac{1}{2} \Big) + \\ + \frac{1}{b^2} D_0 B_{k'} \cdot D_0 B_{k''} W_3^{(0)} + \frac{2}{b^2} D_0 D_{k'} \cdot D_0 D_{k''} W_{11}^{(0)} \Big] e^{i(k'+k'')r} d\tau. \tag{6}$$

If we now introduce the quantities f_k generally used for the description of the transverse electromagnetic field which are defined by the equations:

$$D_k = f_k + f_{-k}^+, \quad B_k = [K_0, f_k - f_{-k}^+] \tag{7}$$

(K_0 — is the unit vector in the direction of K), we see that the perturbation energy H' can be divided in two parts. The first part contains the products of the form $f_{k'} f_{k''}$ and $f_{k'}^+ f_{k''}^+$, so that it does not cause any transitions with the probability proportional to time. The second part contains the products of the form $f_{k'}^+ f_{k''}$ and is just responsible for our scattering effect. One sees immediately that the frequency of the scattered light must be exactly equal to the frequency of the incident light, so that the effect can be described as the deflexion of a light quantum in the field of a fixed centre of force. *

After some calculations we can easily deduce from (6) that the differential cross-section for the process of scattering into the element of solid angle $d\Omega$ under the angle Θ to the direction of the incident beam is equal to (we have taken the sum over both polarizations of the scattered light and averaged it over both polarizations of the incident light):

$$d\Phi = 8\pi^4 \frac{\alpha^6}{\lambda^4} [(p + q\cos\Theta + q' \sin^2\Theta)^2 + (p\cos\Theta + q + p' \sin^2\Theta)^2]\, d\Omega \tag{8}$$

---

* If D_0 does represent not the field of a point charge, but the homogeneous field of a plate condenser, the energy H' can be taken into account quite exactly. In this case, the non-linearity is manifested by the fact that in the presence of the field D_0 the space becomes optically anisotropic. This anisotropy can be caracterized by two tensors of the «dielectric constant» and «magnetic permeability», their components being given by the formulae:

$$(\varepsilon^{-1})_{ij} = 2W_1^{(0)} \delta_{ij} + \frac{4}{b^2} D_{0i} D_{0j} W_{11}^{(0)}, \quad (\mu^{-1})_{ij} = 2W_2^{(0)} \delta_{ij} + \frac{2}{b^2} D_{0i} D_{0j} W_3^{(0)}.$$

where the following abbreviations are used

$$\alpha = \sqrt{Z} \cdot \rho, \quad \rho = \sqrt{\frac{e}{b}}, \tag{9}$$

$$p = \int_0^\infty r^2 \left[2W_1^{(0)}\left(\frac{1}{r^4}\right) - 1 \right] \frac{\sin \beta r}{\beta r} dr + 4 \int_0^\infty \frac{1}{r^2} W_{11}^{(0)}\left(\frac{1}{r^4}\right) \frac{\sin \beta r - \beta r \cos \beta r}{(\beta r)^3} dr, \tag{10_1}$$

$$q = \int_0^\infty r^2 \left[2W_2^{(0)}\left(\frac{1}{r^4}\right) - 1 \right] \frac{\sin \beta r}{\beta r} dr + 2 \int_0^\infty \frac{1}{r^2} W_3^{(0)}\left(\frac{1}{r^4}\right) \frac{\sin \beta r - \beta r \cos \beta r}{(\beta r)^3} dr, \tag{10_2}$$

$$p' = 4 \int_0^\infty \frac{1}{r^2} W_{11}^{(0)}\left(\frac{1}{r^4}\right) \left[\frac{\sin \beta r}{\beta r} - 3 \frac{\sin \beta r - \beta r \cos \beta r}{(\beta r)^3} \right] dr, \tag{10_3}$$

$$q' = 2 \int_0^\infty \frac{1}{r^2} W_3^{(0)}\left(\frac{1}{r^4}\right) \left[\frac{\sin \beta r}{\beta r} - 3 \frac{\sin \beta r - \beta r \cos \beta r}{(\beta r)^3} \right] dr, \tag{10_4}$$

and

$$\beta = 4\pi \frac{\alpha}{\lambda} \sin \frac{\Theta}{2}. \tag{11}$$

In particular, when

$$\lambda \gg 4\pi\alpha \tag{12}$$

(i. e. when λ is large in comparison with the linear dimensions of the region in which $(D_0) > b$ or $\sim b$), the dependence of $d\Phi$ on the wave length of the incident light and the value of the charge Ze is the same for all W. In this limiting case we can namely put in all the integrals (10) simply $\beta = 0$, what gives

$$p = \int_0^\infty r^2 \left[2W_1^{(0)}\left(\frac{1}{r^4}\right) - 1 \right] dr + \frac{4}{3} \int_0^\infty \frac{1}{r^2} W_{11}^{(0)}\left(\frac{1}{r^4}\right) dr;$$

$$q = \int_0^\infty r^2 \left[2W_2^{(0)}\left(\frac{1}{r^4}\right) - 1 \right] dr + \frac{2}{3} \int_0^\infty \frac{1}{r^2} W_3^{(0)}\left(\frac{1}{r^4}\right) dr \tag{13}$$

and $p' = q' = 0$. Introducing this into (8) we find

$$d\Phi = 8\pi^4 \frac{\alpha^6}{\lambda^4} \left[(p^2 + q^2)(1 + \cos^2 \Theta) + 4pq \cos \Theta \right] d\Omega \tag{14}$$

and

$$\Phi = \frac{128\pi^5}{3} Z^3 \frac{\rho^6}{\lambda^4} (p^2 + q^2). \tag{15}$$

2. Let us assume — of course only in order to have before our eyes a concrete example — that W is given by the first Born's expression

$$W = \sqrt{1 + \frac{D^2 + B^2}{b^2} + \frac{D^2 B^2}{b^4}} - 1. \tag{16}$$

Then we obtain from (14)

$$d\Phi \frac{2\pi^4}{9} \gamma^2 \frac{\alpha^6}{\lambda^4} [25(1 + \cos^2 \Theta) - 48 \cos \Theta] d\Omega, \tag{17}$$

where

$$\gamma = \int_0^\infty \frac{dr}{\sqrt{1 + r^6}} = 1.8541. \tag{18}$$

Hence, remembering that according to Born

$$\rho = \frac{2}{3}\gamma\frac{e^2}{mc^2}$$

we have

$$\Phi = \frac{200\pi}{3}\left(\frac{4\pi\gamma^2}{9}\right)^4 Z^3\frac{1}{\lambda^4}\left(\frac{e^2}{mc^2}\right)^6. \quad (19)$$

On the second version of Born's theory

$$W = \sqrt{1+\frac{D^2+B^2}{b^2}+\frac{D^2B^2-(DB)^2}{b^4}}-1, \quad (20)$$

therefore

$$d\Phi = \frac{2\pi^4}{9}\gamma^2\frac{\alpha^6}{\lambda^4}[13(1+\cos^2\Theta)-24\cos\Theta]\,d\Omega, \quad (21)$$

$$\Phi = \frac{104\pi}{3}\left(\frac{4\pi\gamma^2}{9}\right)^4 Z^3\frac{1}{\lambda^4}\left(\frac{e^2}{mc^2}\right)^6. \quad (22)$$

We see that, thanks to the presence of the term containing $\cos\Theta$, the distribution of intensity of the scattered light has a sharp asymmetry relatively to the plane perpendicular to the direction of the incident light. The character of this asymmetry depends strongly on the form of the function W. The order of magnitude of the total cross-section does not, however, change appreciably when we pass from one form of the non-linear theory to another.

Numerical calculations show that for $\lambda \sim 4\pi\alpha$ the total cross-section obtained from the «exact» expression (8) is 2—3 times smaller than that calculated from (15). In general, the increase of total scattering for short waves must go according to (8) noticeably slowlier as it would follow from the λ^4-law.

3. If all the fields with which we have to deal are weak, the expression (1) can be written in quite a general form and contains only two arbitrary constants:

$$W = \frac{D^2+B^2}{2b^2}+\sigma_1\frac{(D^2-B^2)^2}{b^4}+\sigma_2\frac{(DB)^2}{b^4}. \quad (23)$$

The introduction of this expression into the above formulae leads to divergent results (this is analogous to the fact that the self-energy of the point charge calculated from (23) diverges in the non-linear correction terms still stronger than in the first term). The examples given above show, however, that if we introduce (23) into (10) or (13) and carry out the integration not from 0 to ∞ but from 1 to ∞ (i. e. if we ascribe to our charge an effective «radius» of the order of α), the result obtained is in general features the same as that obtained with the help of the corresponding «exact» W*. Thus, for general orientation it is not useless to quote the expression for $d\Phi$ obtained in such a way by arbitrary σ_1 and σ_2:

$$d\Phi = \frac{32\pi^4}{9}\frac{\alpha^6}{\lambda^4}\{[100\sigma_1^2+(\sigma_2+6\sigma_1)^2](1+\cos^2\Theta)+40\sigma_1(\sigma_2-6\sigma_1)\cos\Theta\}\,d\Omega \quad (24)$$

and

$$\Phi = \frac{512\pi^5}{27}Z^3\frac{\rho^6}{\lambda^4}[100\sigma_1^2+(\sigma_2-6\sigma_1)^2] \quad (25)$$

---

* Therefore, our results do not substantially depend on the distribution of charge within the nucleus—if we assume that the real linear dimensions of the nucleus are not markedly greater than α.

(both these expressions are valid for $\lambda \gg 4\pi\alpha$).

According to Heisenberg and Euler we have

$$\sigma_1 = -0.49; \quad \sigma_2 = 7\sigma_1.$$

In this case the angle distribution is determined by the factor

$$101\,(1 + \cos^2\Theta) + 40\cos\Theta,$$

i. e. the greatest part of light is scattered under the angles $\Theta < 90°$. For Φ we find

$$\Phi \approx 1.4 \cdot 10^5 Z^3 \frac{\rho^6}{\lambda^4},$$

what leads—if we put $\rho = \frac{e^2}{mc^2}$—to the numerical values of the same order of magnitude as those obtained with Born's W.

4. If we consider the problem of the interaction of a radial electrostatic field and the field of a light wave from the standpoint of Dirac's theory of the positron, the preceding treatment can be apllied only in a certain limiting case, i. e.

a) when we have to deal not with a point charge, but with a charged sphere, the radius of which is larger than the Compton's wave length and the total charge has such a value that the field in each point is $\ll \frac{m^2c^3}{he}$;

b) when the wave length λ is larger than the Compton's wave length.

The extrapolation of the obtained results in the case of $\lambda < \frac{h}{mc}$ and the Coulomb field of a point charge is from this standpoint unlawful. It seemed to us, however, not useless to see, at first, what would give the simplest theory because here we have to do with a scattering phenomenon which can be really observed and therefore can serve for comparison of results of different theories.

For $\lambda = 4.7$ $X.E$ and $Z = 82$ the above formulae give $\Phi \sim 10^{-24}$ cm^2, what does not contradict the observations of Meitner (1) and of Bothe and Horn (2) on the hard component of the radiation scattered by lead. The angle distribution derived with help of Born's expression for the energy density is, however, in sharp contradiction with the observation of Bothe and Horn.

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REFERENCES

1 ZS. f. Phys., 84, 137 (1933). 2 ZS. f. Phys., 88, 683 (1934).

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PHYSICAL CHEMISTRY

ON PSEUDOEXTRACTION AND ON SOME SPECIFIC PROPERTIES OF FILMS OBTAINABLE THEREBY

By L. I. BELAJEV

(Communicated by V. A. Kistiakovskij, Member of the Academy, 2. III. 1937)

The recent development of Acad. V. A. Kistiakovskij's film theory involves a detailed study of the process of formation and of the properties of films. Among isolated film-like formations, a special place is occupied by films obtainable at the interface of liquid phases, more particularly by the «pseudoextraction» method. In his investigation «On the Problem of Metastable Flotation» ([1]), Acad. V. A. Kistiakovskij has examined these films with particular care and suggested a novel and original approach to the problem.

On the suggestion of Acad. V. A. Kistiakovskij, we started an investigation (at first only a qualitative one) both of the process of film formation in itself and of some specific properties of films. Apart from a certain outward analogy to a true film, these films has also arrested our attention because they had been studied rather inadequately and unsystematically, as may be seen from the literature available on the question (cf. list given below).

In order to obtain the film, we generally used some inorganic hydrosols (or suspensions) and caused them to emulsify with some liquid that did not mix with water ([2], [3]). Upon coalescence of the emulsion, the disperse phase collects at the interface of the liquids and forms what we shall herein called, for brevity's sake, a film. A necessary preliminary to pseudoextraction is the astabilization of the sol by the action of electrolytes, dehydrating substances, etc. The sensibilization, coagulation or dehydration of the sol (e. g. sols of gold, of ferric hydroxide after Graham, of Prussian blue, of copper ferrocyanide) contribute to the formation of a film. Thus previous to pseudoextraction, the disperse phase of the sol must lose its «secondary stability» ([4]) and, at the same time, lower the degree of its dispersion.

This would suggest that the mechanism of film-formation is similar in nature to flotation (more particularly, to the Elmore's Bulk-oil flotation process) which, according to modern ideas, is connected with wetting phenomena ([5]). As regards the substances used by us, the main point was the wetting, or non-wetting, of the disperse phase by the non-aqueous liquid phase of the emulsion. In the case of suspensions, preliminary coagulation

was usually found to be unnecessary, the phenomenon of film formation at the interface of the liquid phases being solely due to wetting factors, primarily to whether the disperse phase was a lyophile or a lyophobe one. We obtained macroscopic films from the following suspensions: metallic aluminium, magnesium, coal, ferric oxide, oxide of mercury, aluminium oxide, copper oxide. Since these substances are capable of film formation, it may be inferred that their surfaces are better wetted by the non-aqueous and less polar phase than they are by water; to all appearances, they are hydrophobic. This is further confirmed by the fact that so signally a hydrophile powder as SiO_2 forms no film.

We used as «pseudoextractors» (i. e. as the non-aqueous phase) isoamyl alcohol, ether, benzol, vaseline oil. The use of the latter hinders «pseudo-extraction». The use of ether or benzene gives rise to the formation of unduly expanded films. We ultimately chose isoamyl alcohol as giving less expanded films which are more suitable for observation. A film can also be obtained on the interface with mercury; for this purpose, the latter is emulsified in a sol (e. g. in a gold sol), amalgamation taking place simultaneously. From the surface of the mercury this film may be transferred to the water/isoamyl alcohol interface by emulsifying in a water /mercury/isoamyl alcohol system.

The filling up of the sol/benzene interface upon coalescence takes place at the moment when the separation into layers is at an end, and when the films almost instantaneously cover up the interface.

The rate of pseudoextraction processes depends on the concentration of the sol, film formation being retarded in both highly concentrated and highly diluted sols. The high dispersion of the gold sol greatly retards «pseudo-extraction». The area occupied by the film depends on the degree of dispersion, being symbatic with the latter: the higher the dispersion of the sol, the greater the area.

In order to obtain films at the interface of liquid phases, we also used the «electrophoretic pseudoextraction» method developed by ourselves. This latter is as follows. The hydrosol is poured into a flat dish upon a liquid that will not mix with it. Immersed into this system are platinum electrodes, which pierce both liquid layers. If a direct electric current (e. g. a 220 V, 2 mA current) is passed through the circuit, a cataphoresis of the disperse phase of the sol takes place, causing it to coagulate near the oppositely charged electrode. The coagulant collects at the interface of the liquids as a film, being apparently floated by the gas bubbles evolved on the platinum, and forms a film.

As a rule, the colour of the films is that of the sol, or its coagulate,—in a passing light. Thus, gold films, when viewed in a reflected light, had the colour and brilliancy of gold, but looked blue or violet if examined in a passing light. The films were as a rule more or less transparent.

In respect of microscopic structure, all the films examined by us may be subdivided into three types:

I. Lamellar films: built up of discrete formations (lamellae) of scale-like appearance and varying size, their diameters being of the order of 0.05 to 0.5 mm (e. g. gold films). The lamellae are distinct parts of the film, circumscribed by definite boundaries, although they are not, properly speaking, the primary elements of the film.

II. Laminar films built up of continuous plates reminiscent of the epidermis (as, for instance, the films obtained by Graham's method out of a ferric hydroxide sol).

III. Films of an intermediate type, obtained, according to conditions, either after type I or after type II.

The films obtainable from the lyophobe sols of powders and suspensions are as a rule of type I. As to lyophile disperse systems, the films formed were most frequently of the second (or third) type.

A lamellar film assumes the shape of the surface of the walls of the containing vessel. Films formed in a cylinder have the shape of a «bag», the base of which rests on the interface of the phases; the edges of the «bag» are irregular. In the case of lamellar films, minute holes sometimes appear. Not infrequently, laminar films are folded and exhibit multilamination.

According to our microscopic observations, lamellar films (e. g. those of gold) lie above the interface of the phases in the non-aqueous phase; in other words, a layer of the non-aqueous phase is formed between the hydrophobic film and the true interface. This is also proved by an experiment consisting in the isolation of the film: by pouring in ethyl alcohol, the lamella are readilly detached from the water/ether interface and transferred into the ether phase as tubes not unlike Evans's isolated film ([6]). Another proof is based on an electrophoresis experiment: if a drop of the non-aqueous phase (isoamyl alcohol) is placed between two electrodes, electrophoresis does not cause the lamellae that are at the interface of the liquid phases to transcend the boundaries of the drop (a 400 V, 5 mA direct current). The hydrophilous type films (the laminar films, e. g., those of ferric hydroxide) face as a rule the aqueous phase.

The structure of the film and its total area depend mainly on the quantity of the disperse phase, on the presence of electrolytes and non-electrolytes (more especially of surface-active substances), on the ratio of the aqueous to the non-aqueous phase, and on the diameter of the film «bag».

The unusual position of the films accounts for their peculiarities. Some of them are described below. The «inversion» of the film occurs under the action of some polar liquids or surface-active substances. If an excess of ethyl alcohol is added to the ether (lying over the gold film), the lammellae pass from the interface into the bulk of the non-aqueous phase: an excess of ether causes the lamellae to leave the bulk of the non-aqueous phase and to return to the liqud/liquid interface ([7]). The introduction of a solid sodium oleate onto the liquid/liquid interface causes the lamellae to be suddenly forced up along the walls of the cylinder; as a result of subsequent emulsifying, the gold phase passes from the liquid/liquid interface into the aqueous phase.

The fact that electrophoresis (with platinum electrodes) causes the gold film to contract indicates that the film is expanded ([8]). Any large lamellae introduced on the clear liquid/liquid interface disaggregate into a great number of smaller lamellae. (A similar disintegration process is sometimes observed at the outset of the electrophoresis, when smaller lamellae become detached from the larger ones and drift towards the electrode, i. e. when an electrodispersion of lamellae takes place.) An addition of certain polar liquids (ethyl alcohol, ether) at the water/isoamyl alcohol interface or an increase in the amount of the non-aqueous phase result in an increase of the area of the film, i. e. in an expansion of the latter. Contractions of the gold film due to addition of ethyl alcohol to the non-aqueous phase (benzene, ether) have also been observed. Under the action of powdered coal, the lamellae of gold and aluminium films form with the coal a more compact film due to mutual sorption. Another phenomenon of the same order is

the fact that isoamyl alcohol draws together powdered palmitic acid (or aluminium) scattered over the surface of the water.

An interesting phenomenon (and a fully reproducible one) is observed in lamellar films at the liquid/liquid interface. If air is insufflated from above (e. g. through a capillary tube), the lamellae of gold are attracted to its point of action. If, on the contrary, air is pumped out, the lamellae are driven away from that spot. The presence of a non-aqueous phase is indispensable to bring this about. If a stream of hydrogen is passed through the interface, the motion of the lamellae away from the stream becomes noticeable. This phenomenon has the outward appearance of electrophoresis and may therefore be termed «pneumophoresis» or «pneumotropism».

Another kind of tropism (or phoresis) is due to the action of water: the pumping out of water from under the film with capillary siphon causes the lamellae to move towards the mouth of the stream; the pumping in of water does, on the contrary, drive them away from that spot. If isoamyl alcohol is dropped on to the liquid/liquid interface out of a microburette, the lamellae are found to move towards the spot on to which the drops fall. The dipping of a moistened glass stick at the liquid/liquid interface causes the lamellae to move away from the stick. These facts suggest the existence of a «lyophoresis» (or «lyotropism»), and, more particularly, «hydrophoresis». They may possibly be due to the formaton of surface solutions ([8]).

Another «phoresis», due to the action of surface tension, wetting and adsorption forces, may also be mentioned. If gold lamellae are allowed to stand, they gradually move away from the interface plane of the liquid phases and collect at the walls of the (glass or porcelain) vessel, ascend along the walls, and are finally sorbed by the latter. Similar to this is the rise of gold lamellae through filter paper, if the latter is dipped into the film ot the liquid/liquid interface. This phenomenon may be termed «capillaryphoresis» or «sorbentophoresis».

Other phenomena observed by us at the liquid/liquid interface («thermophoresis», «photophoresis», etc.) are not described in this paper.

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REFERENCES

[1] В. А. Кистяковский, ДАН, 419 (1930). [2] Zsigmondy-Thiessen, Das kolloide Gold, Leipzig (1925). [3] Reinders, Koll.-ZS., **13**, 235 (1913). [4] Verwey, Chem. Rev., **16**, 363 (1935). [5] П. А. Ребиндер, Липец, Римская, Таубман, Физико-химические основы флотации, Москва (1932). [6] Эванс, Коррозия металлов (1930). [7] Janek u. Schmidt, Koll.-ZS., **47**, 336 (1904). [8] A. Marcelin, Koll.-Beihefte, **38** (1933). [9] Metkalf, ZS., f. ph. Ch., **52**, 1 (1905). [10] Filme und Fäden, Koll.-ZS., **61**, H. 2 (1932). [11] Мокрушин, Журнал физической химии, **5** (1935) и след. [12] Ramsden, ZS. f. ph. Ch., **47**, 336 (1904) [13] Хеджс, Защитные пленки на металлах (1934).

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CHEMISTRY

EQUILIBRIUM DEHYDROGENATION OF *n* -BUTYLENES TO BUTADIENE $n\text{-}C_4H_8 \leftrightarrow C_4H_6+H_2$

By M. I. DEMENTJEVA, A. V. FROST and E. K. SEREBRIAKOVA

(Communicated by V. E. Tiščenko, Member of the Academy, 2. III. 1937)

The determination of the equilibrium dehydrogenation of *n*-butylenes to butadiene is of great importance for the direct production of butadiene from butylenes. The possibility of this dehydrogenation is to be seen from the experiments of Frey and Huppke (1). These authors studied the equilibrium constants of *n*-butanes and found that, during dehydrogenation of butane over chromium oxide as a catalyst, not only butylenes, but also small yet noticeable amounts of butadiene were formed. In our study, we also used chromium oxide as a catalyst. The experiments were carried out in a flow system. The reactor was a U-formed tube of Pyrex glass, placed in a lead bath. The reaction temperature was measured by a thermocouple imbedded in the catalyst. The analyses of the reaction products were carried out in the following way: butadiene was determined by absorption in melted maleic anhydride; *n*-butylenes, by absorption in 84% sulphuric acid; ethylene was absorbed by bromine; hydrogen and saturated hydrocarbons were determined by combustion over copper oxide. In some experiments, the described method of analysis was checked up by fractional distillation of the gas, each fraction being then analysed separately (Table 1). The results of fractional distillation have shown that the usual method of analysis does not involve any appreciable errors, for the exit gas contained but a very small amount of propylene.

In some experiments, the determination of butadiene was checked up by the tetrabromide method.

The molecular weight of liquid products was determined by the cryoscopic method. In experiments that were carried out at temperatures below 480° C, the amount of liquid products was very small and therefore was not taken into consideration.

In experiments carried out in the presence of the catalyst used, measurements may be made at temperature ranging from 480 to 534° C. At these temperatures, equilibrium gets established to within the errors of analysis. At lower temperatures, the reaction runs too slowly for the equilibrium to be reached. At temperatures over 540° C, reactions of homogeneous polymerization occur, these reactions leading to the formation of a great amount of liquid products hindering the study of the predominating reaction.

Table 1

Temperature 480°C.

Composition of Gas before and after the Experiment in Volume Percentages

	Inlet gas	Exit gas
H_2	10.0	12.2
CH_4	11.1	3.9
C_2H_6	0.0	6.1
C_3H_8	0.0	3.5
C_4H_{10}	0.0	1.0
C_2H_4	6.0	0.4
C_3H_6	0.0	0.1
n-C_4H_8	66.7	62.4
C_4H_6	6.2	4.2
C_5Hm	0.0	6.2

Changes of contact time and of gas composition did not cause any systematical changes in the equilibrium constants calculated from experimental data. The considerable variation of the calculated equilibrium constants is due to errors of analysis. The error in the determination of butadiene could amount to 1%. The amount of butadiene in the off gas being very small, this error resulted in appreciable differences between the various experiments. The second cause of errors is the process of butadiene polymerization which leads to a steady decrease of the butadiene concentration and prevents the reaction under investigation from reaching an exact equilibrium.

In order to obviate these errors in the determination of equilibrium, we tried to conduct experiments in a vacuum. It was believed that vacuum would give better results, because the equilibrium concentration of butadiene in a vacuum is higher than at atmospheric pressure; on the other hand, processes of polymerization ought to be of less importance in a vacuum. However, using chromium oxide as a catalyst, as had been done at normal pressure, we did not get more accurate results. At temperatures of 480 and 534° C, 100 mm Hg, and time contact of from 1 to 10 sec. over chromium oxide, a vigorous decomposition of hydrocarbons to carbon, hydrogen and lower paraffins was observed to occur. The ratio, calculated from the equation $K_p = \frac{P_{H_2} \cdot P_{C_4H_6}}{P_{n\text{-}C_4H_8}}$, varied, in dependence upon the composition of the inlet gas and time contact, for 480° C from 0.0019 to 0.0058 and for 534° C from 0.005 to 0.028, these values being rather understated. In order to obtain more accurate data, it is necessary to find such a catalyst as would catalyze the decomposition reactions in a vacuum to a smaller extent.

The average equilibrium constants for temperatures of 480 and 534° C were calculated from the data of several experiments. These data, as well as the upper limits of the equilibrium constants for temperatures of 404 and 444° C, are shown in Table 2.

Inasmuch as equilibrium at temperatures of 404 and 444° C was not reached and the reaction at these temperatures was studied only with respect to the gas mixtures containing an excess of hydrogen and butadiene, the data for these temperatures should be regarded merely as maximum values. It will be seen later on that the real data are a little lower.

Table 2

Results of the Study of the Equilibrium of the Reaction $n\text{-}C_4H_8 \leftrightarrow C_4H_6 + H_2$

T°C	T°K	Kp Exper. *	Exper. lg K_p	Calcul. lg K_p	Calcul. K_p
404	677	<0.006	<-2.2	—2.85	0.0014
444	713	<0.008	<-2.1	—2.87	0.0043
480	753	0.013 ± 0.005	—1.89	(—1.89)	(0.013)
534	807	0.048 ± 0.015	—1.32	(—1.32)	(0.048)

* Pressure in atmospheres: $Kp = \frac{P_{H_2} P_{C_4H_6}}{P_{n\text{-}C_4H_8}}$

The equilibrium constants given in Table 2 for 480 and 534°enter into the equation:

$$\lg K_p = -\frac{6\,413}{T} + 6.63 \pm 0.13.$$

This equation gives the heat effect of the reaction:

$$Q = -29.0 \text{ kilocal./mole.}$$

Taking into consideration the errors of the equilibrium constants in Table 2, the error of this value should be estimated at about 10—12 kilocal/mole. Direct measurements of the heat of hidrogenation of butadiene and normal butylenes by Kistjakowsky and his collaborators ([2]), gave for 82° C the following values. The heat of hydrogenation of butadiene to α-butylene is 26 700 cal./mole; to β-*cis*-butylene, 28 500 cal./mole; to β-*trans*-butylene, 29 400 cal./mole. The value calculated by us from the equilibrium data is in agreement with these values.

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REFERENCES

[1] F. E. Frey, W. F. Huppke, Ind. Eng. Chem., 25, 54 (1933) [2] G. B. Kistjakowsky, I. R. Ruhoff, H. A. Smith, W. E. Vaughen, Journ, Am. Chem. Soc., 57, 876 (1935); 58, 146 (1936).

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GENETICS

ANALYSIS OF THE ALLOBALANCE OF A CHROMOSOME BY TWO SIGNALS. UTILIZATION OF SEVERAL QUANTITATIVE CHARACTERS SIMULTANEOUSLY

By A. S. SEREBROVSKIJ, Corresponding Member of the Academy

We have elsewhere shown that in a genetical analysis of quantitative characters, new perspectives may be opened by analysing several different quantitative characters simultaneously. In the present, we intend to discuss this question as regards an analysis of a chromosome by two signals S_1 and S_2. It is not the complete balance of the chromosome that we analyse, but the difference between the balance of two homologous chromosomes found in a heterozygous individidual—that is to say, what we call the «allobalance».

We designate the genes which influence character **A** by A_1, A_2... and the genes influencing character **B** by B_1, B_2,... Suppose, for example, we have a heterozygous female *Drosophila* or male silk moth and cross it with a homozygous male (resp. female) analyser. Above each gene, taking an arbitrary example, we write the force of its action:

$$\overset{+2\ \ +4\ \ \ \ -2\ \ \ \ +3}{\underset{+3\ \ \ \ +4}{\frac{S_1\ B_1\ B_2\ A_1\ B_3\ A_2\ B_4\ S_2}{s_1\ b_1\ b_2\ a_1\ b_3\ a_2\ b_4\ s_2}}}\ \text{♀} \times \frac{s_1\ b_1\ b_2\ a_1\ b_3\ a_2\ b_4\ s_2}{s_1\ b_1\ b_2\ a_1\ b_3\ a_2\ b_4\ s_2}\ \text{♂}$$

From the offspring of such a cross, we take goup $S_1\, s_2$, composed of individuals distinguishable by signals S_1 and s_2. These individuals will be either single crossings over or else triple crossings over in the section S_1—S_2. We will record only the single ones possible in this structure and will determine the value of characters **A** and **B** for each one.

Place of crossing over	Structure	Value of character	
		A	**B**
1	$S_1 b_1 b_2 a_1 b_3 a_2 b_4 s_2$	0	0
2	$S_1 B_1 b_2\ {}_1 b_3 a_2 b_4 s_2$	0	2
3	$S_1 B_1 B_2 a_1 b_3\ {}_2 b_4 s_2$	0	6
4	$S_1 B_1 B_2 A_1 b_3\ {}_2 b_4 s_2$	3	6
5	$S_1 B_1 B_2 A_1 B_3 a_2 b_4 s_2$	3	4
6	$S_1 B_1 B_2 A_1 B_3 A_2 b_4 s_2$	7	4
7	$S_1 B_1 B_2 A_1 B_3 A_2 B_4 s_2$	7	7

Plotting the seven points we obtained on the field **A B** (Fig. 1).

We have extended a broken strip through the 7 plotted points. In plotting this strip, we might even not know the ordinal numbers of the 7 points, but the strip is the only one possible if, in plotting it, one be guided by the consideration 1) that the angles (flexures) should pass through the points and do not lie in an empty place, and 2) that the sections of the strip should be parallel with either axis **A** or axis **B**.

The following properties of this strip ought to be noted:

1. The distance between points is equal to the forces of the genes; thus, the distance between points *1* and *2*, equal to 2, is the force of gene B_1 (=+2); the distance between points *2* and *3* (=+4) is the force of gene B_2, etc.

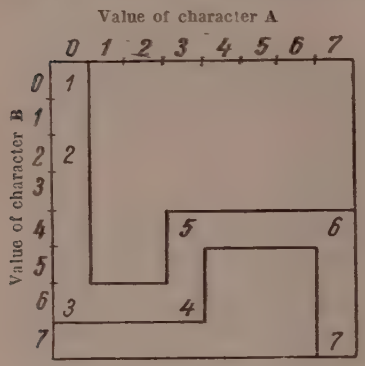

Fig. 1.

2. The fact that sections between neighbouring points are parallel with axis **A** points to the influence of the corresponding gene on character **A**; the parallelism of sections with the axis **B** points to their influence on character **B**. For example section *1—2* is parallel with axis **B** and corresponds to gene B_1; section *3—4* is parallel with axis **A** and corresponds to gene A_1, and so forth.

3. The order of the sections corresponds to the order of distribution of the genes in the chromosomes.

4. The number of angles in the strip corresponds to the number of «changes» of *A* and *B* in the order of their distribution along the length of the chromosome. In our instance, the number of places of the «changes» are the followings:

$$S_1 \quad B_1 \quad B_2 \quad A_1 \quad B_3 \quad A_2 \quad B_4 \quad S_2$$

$$\underbrace{\uparrow_1 \quad \uparrow_2 \quad \uparrow_3 \quad \uparrow_4}_{\text{Changes}}$$

5. Frequency of points expresses the frequency of crossing over between genes. If, for example, point *5* is met with in the empirical material twice as often as point *4*, it follows that the distance between the 3d and the 4th genes (B_2, and A_1) is less than the distance between the 4th and 5th genes (A_1 and B_3), although not exactly 2 times less, but in the proportion

$$c_3\,(1-c_4):(1-c_3)\;c_4,$$

where c_3 and c_4 are values of the crossing over in the sections *3* and *4*, respectively expressed in parts of a unity.

Now let us examine the advantages of investigating two characters simultaneously. If we paid attention to only character **A**, then among $S_1 s_2$ individuals the character value would be 0, 3 and 7. From this we might conclude that in section S_1—s_2 there are two genes influencing character **A**, and that these three values corresponded to genotypes $a_1\,a_2$, $A_1 a_2$ and $A_1 A_2$ (1). But no conclusion would be possible about the order of distribution of the genes. We would have 5 values with respect to character **B**—0, 2, 4, 6 and 7, and might infer about 4 genes, but it would also be impossible to draw any conclusion about their distribution. But having laid

them out on a plane, we are enabled to arrive at such a conclusion. Here the distribution of genes B among genes A is of especial importance and, conversely, of genes A among genes B. From this point of view, the more independent characters we may investigate simultaneously the clearer the picture we obtain. If genes A_1 and A are situated in the chromosomes, then the gene exerting an influence on character **B** can not appear between them (at least in the heterozygous state which is necessary for such an analysis). But, on the other hand, gene C_k, exerting an influence on character **C**, may prove to be situated here, or gene D_k, influencing character **D**, and so forth.

This method of analysis has the following complications. Among S_1s_2 individuals, triple crossings over, each of which has its own value of charac-

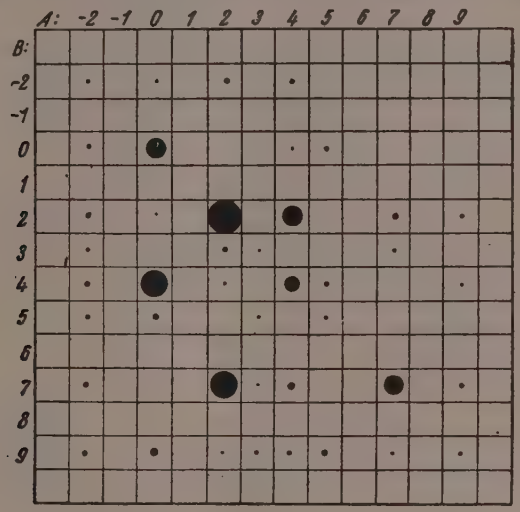

Fig. 2.

ter **A** and **B**, may be met, aside from single crossing over. Consequently, there appear on our field yet other supplementary points, which may complicate the reading of the figure and the drawing of the «strip». True, triple crossings over will be met with much less frequently than single crossings over. For example, if the distance between the genes is 10%, single crossings over will be met with more than 100 times as often as the triple. But if some of the intervals between genes are very small and others very large, then some of the single crossings over will be met as often as some of the triple, or even less often.

If the distance between signals is small, triple crossings over may be reduced to zero. But in such a case, there arises a second complication. Genes influencing **A** and **B** and situated to the left of the left signal and to the right of the right signal may be present. Then, there may appear in the group of S_1s_2 individuals some of which have a double crossing over: one passing between the signals and the second, somewhere outside of the section S_1—s_2. This brings it about that every «point» in our field proves to be doubled, tripled, and so on. And that, in turn, doubles, triples, and so on the whole strip and complicates also the charting of it. The use of three or more signals might help overcome this difficulty, but that is outside the subject of the present paper.

Complications may also be called forth by genes situated in other chromosomes and this also would double, triple, and so on our points. This complication may be overcome, however, by determining the value of the character for a whole group of individuals and not for just one. For example, we may obtain male *Drosophila* or female silk moths from the initial crossing and cross each of them with female (resp. male) analysers. Since crossing over does not occur in the male *Drosophila* (or in the female silk moth), so all the S_1s_2 individuals resulting from such a male will retain the S_1s_2 chromosome unchanged. With a large amount of material, it would be possible to study the value of characters **A** and **B** freed from the influence of other chromosomes which in this or the following generation might be eliminated.

To illustrate this, we calculate the frequency of different single and triple crossings over in one case in which the two signals were situated at the ends of the chromosomes at a distance of 100 morganides from each other. For simplicity, coincidence was taken everywhere as equal to unity, which increased the frequency of triple crossing over. The frequencies of the different categories are represented in Fig. 2 by circles of different diameters. Although a rather large number of «points» are to be seen in the figure, certain of them are conspicuous by their size. There are 7 such points. Let us try to draw a «strip» through them. From the extreme upper left (*0*; *0*), the strip can, it is apparent, pass only downwards to (*0*; *4*). Further there is a large point only on the horizontal to the left (*4*; *4*). From the latter there is a notable turn upwards to (*4*; *2*) and further on to the largest point of all (*2*; *2*). From there, the strip may be continued only along the vertical downwards to (*2*; *7*), and further to the last large point (*7*; *7*).

This solution we write thus:

Sections (points)	Genes	Force	Area of circle	Approximate distance in morganides
	S_1			
1			37	9.1
	B_1	+4		
2			83	20.5
	A_1	+4		
3			17.5	4.3
	B_2	−2		
4			37	9.1
	A_2	−2		
5			110	27.4
	B_3	+5		
6			83	20.5
	A_3	+5		
7			37	9.1
	S_2			
			404.1	100.0

In this approximate solution the smaller distances are somewhat underreckoned and the larger ones somewhat exaggerated. According to the condition of the problem, the distances were 5, 10, 20 and 25 morganides. The distances may be more accurately found by solving a system of equations:

$$c_1(1-c_2)(1-c_3)(1-c_4)(1-c_5)(1-c_6)(1-c_7) = 0.37,$$
$$(1-c_1)c_2(1-c_3)(1-c_4)(1-c_5)(1-c_6)(1-c_7) = 0.83,$$
$$(1-c_1)(1-c_2)c_3(1-c_4)(1-c_5)(1-c_6)(1-c_7) = 0.175.$$

The unknowns c_1, c_2,..., c_7 are the distances between genes or the lengths of the sections in terms of the value of the crossing over (expressed in parts of a unity).

Let us return now to Fig. 2. We chose the 7 large points for our solution. But at (*0*; *9*) there is still another point, the largest of the small ones. We will try to include it in our solution, drawing the strip through points (*0*; *0*), (*0*; *4*), (*0*; *9*), (*4*; *9*) (*4*; *4*), (*4*; *2*), (*2*; *2*), (*2*; *7*), and (*7*; *7*). This solution we write thus:

Sections (points)	Genes	Force	Area of circle	Approximate distance in morganides
	S_1			
1			37	9.0
	B_1	+4		
2			83	20.1
	B_2	+5		
3			7	1.7
	A_1	+4		
4			1	0.3
	B_3	−5		
5			17.5	4.2
	B_4	−2		
6			37	9.0
	A_2	−2		
7			110	26.6
	B_5	+5		
8			83	20.1
	A_3	+5		
9			37	9.0
	S_2		412.5	100.0

Comparing this solution with the foregoing, we find two extra genes in it— B_2 (+5) and B_3 (—5), which are separated from each other by a distance of 2 morganides. Such a coincidence as that two mutually neutralizing genes should be situated right alongside each other, justifies one in regarding these genes as unreal. But it must be borne in mind that the existence of a pair of mutually neutralizing genes alongside each other is possible for any point in the chromosome and can be neither proven nor refuted. Consequently, such a solution can only be admitted when there is weighty evidence for it. In this instance, there is no weighty evidence for the existence of this pair of genes, and this minor point (*0*; *9*) can be referred to the points of triple crossing over. This assumption is easy to test out. If our first solution is correct, then the most frequently met triple crossing over would be the 2, 5, 6 crossing over. It ought to correspond to the genotype $S_1B_1a_1b_2a_2B_3a_3s_2$ and have character values $\mathbf{A}=0$ and $\mathbf{B}=9$.

The frequency of this point (*0*; *9*) ought to be

$$(1-c_1)\,c_2\,(1-c_3)\,(1-c_4)\,c_5c_6\,(1-c_7),$$

or, according to our approximate solution,

$$0.909 \cdot 0.205 \cdot 0.957 \cdot 0.909 \cdot 0.274 \cdot 0.205 \cdot 0.909 = 0.0083 \text{ or } 0.83\%,$$

whereas the rarest single crossing over (3) ought to be encountered with a frequency

$$(1-c_1)\,(1-c_2)\,c_3\,(1-c_4)\,(1-c_5)\,(1-c_6)\,(1-c_7),$$

or, according to our approximate solution,

$$0.909 \cdot 0.195 \cdot 0.043 \cdot 0.909 \cdot 0.726 \cdot 0.795 \cdot 0.909 = 0.0148 \text{ or } 1.48\%.$$

Thus, the area of the circle of frequency of the triple crossing over 2, 5, 6 ought to be approximately two times less than the area of the circle of crossing over 3. In reality, point (*0*; *9*) is just where we find the largest of the small points and it has an area of about 7, that is, about $2^1/_2$ times less than the area of the circle at point (*4*; *4*). The calculations show good agreement so that we ought to regard this point (*0*; *9*) as point of the 2, 5, 6 crossing over and ought not to carry our «strip» through it, that is, we ought to go back to our first solution. It, indeed, corresponds to the condition laid down in the statement of the problem:

	$\overset{+4}{B_1}$		$\overset{+4}{A_1}$	$\overset{-2}{B_2}$	$\overset{-2}{A_2}$		$\overset{+5}{B_3}$	$\overset{+5}{A_3}$	
S_1		10	20	5	10	25	20	10	S_2

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REFERENCES

[1] М. Л. Карп, Зоологический журнал, XV, вып. 1, 26—47 (1936).

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GENETICS

AMPHIDIPLOID SHOOTS OF THE F_1- HYBRID *NICOTIANA TABACUM* L. × *N. SYLVESTRIS* SPEG. ET COMES

By M. F. TERNOVSKIJ

(Communicated by A. A. Sapehin, Member of the Ukrainian Academy of Sciences 4. III. 1937)

Among a large quantity of hybrid sterile plants of the first generation of *N. tabacum* ($n=24$) and *N. silvestris* ($n=12$) there was found in summer 1936 one fertile plant which gave normal pods when freely flowering under the isolator and when crossed with paternal forms. Yet two months later, on the lower part of the stem, two completely sterile shoots were discovered. By its morphological characters, the plant was a typical hybrid of the first generation. The flowers of the fertile shoots were characterized only by somewhat larger dimensions.

Cytological investigations showed that a) the eight investigated rootlets had 36 chromosomes; b) in the metaphase of the homeotypical division, the fertile shoots had 72 chromosomes in both plates; c) the sterile shoots had 36 chromosomes in the metaphase of the homeotypical division. Thus, it would seem that we have to deal here with a chromosomal chimera bearing several amphidiploid shoots in its upper part.

As regards their gonogenesis, the sterile shoots were in no way to be distinguished from the usual digenom hybrids studied by Goodspeed (2), Goodspeed and Clausen (3), and others. Their reduction division follows the *Drosera* type. Twelve chromosomes of *N. sylvestris* conjugate with twelve chromosomes of *N. tabacum* forming twelve bivalents. The other twelve chromosomes of *N. tabacum* remain univalents. They interfere greatly with the reduction division.

The fertile shoots showed an altogether different picture as to their gonogenesis. The metaphase of the first division takes place in one plane. We met only very seldom figures where 1—3 chromosomes had not been included in the equatorial plane. The number of separate bodies in the metaphase of the first division varies from 33 to 36 (the average of 18 calculations gave 34.5 separate bodies). In this our amphidiploid shoots differ from the amphidiploid *N. tabacum* × *N. sylvestris* obtained in F_2 by Egis (1). Rybin (5) found here 28 separate bodies as the most usual number in the metaphase of the heterotypical division. The number of elements in the metaphase of the first division suggests the presence of quadrivalent groups of chromosomes. Only open quadrivalents that do not form a ring were found in the diakinesis. In the metaphase of the first division they are conspicuous by their con-

siderable size. One can sometimes study their structure in microscopic sections stained with acetocarmine and in smears stained with gentian-violet according to Newton.

The anaphase of the first division is mostly regular, but cases of non-simultaneous separation of the chromosomes towards the poles were also to be observed. Sometimes one or two chromosomes do not reach the poles and remain in the plasm. An analysis of 90 cells showed 21.1% of such cases. In the metaphase of the second division, the number of plates with 36 chromosomes was 61.5% (out of 26 cells studied). In the other cases, the number of chromosomes was found to vary from 34 to 38. This shows that the gametes of an amphidiploid must have unequal numbers of chromosomes and that, besides the 72-chromosome plants, there may exist various aberrant types. An analysis of the second anaphase showed no irregularities. The pollen of the amphidiploid shoots is 98.2% filled with plasm. As compared with paternal forms and with the sterile shoots, this pollen is very large-grained. Measurements showed the following dimensions in microns: *N. tabacum*, 33.40; *N. sylvestris*, 33.13; amphidiploid shoots, 42.86; digenom shoots, 16.54. The germination of the pollen of amphidiploid shoots showed an insufficiently high percentage and the pollen tubes grew rather slowly. This explains their comparatively incomplete fertility: the average number of seeds per pod in these shoots was 378, whereas that in the paternal form *N. tabacum* was 2000 and in *N. sylvestris* about 3000.

The way I obtained a fertile form from the *N. tabacum* × *N. sylvestris* hybrid in the first generation is somewhat unusual. I used Randolph's method ([4]) of high temperature effect. In 1935, 4 to 6 days after their pollination by *N. sylvestris*, flower buds of *N. tabacum* were five times exposed to the action of a temperature of 40.4—46.2° in special chambers. The plant in question was discovered in the offspring of two such buds, seeds from which were sown together. By the moment of the exposure, the fertilized egg-cell seems to have formed a considerable number of embryo-cells and the doubling took place in one of them at some later stage of the division of the embryo-cells. This seems to be what caused to originate a plant with cells containing different numbers of chromosomes.

However, it is possible that the amphidiploid chimera arose naturally, independently of the action of temperature factors. Such cases have been observed among the interspecific hybrids *N. tabacum* × *N. glauca* and *N. glutinosa* × *N. tabacum* [Ternovskij ([6])]. However, I consider this possibility less likely, inasmuch as notwithstanding numerous experiments made by various investigators and the fact that I studied many thousand F_1 plants (in 1936 alone, 4724 plants were under observation), an amphidiploid in the first generation was discovered for the first time and only in the offspring from buds exposed to the action of high temperature.

Received
4. III. 1937.

REFERENCES

[1] С. А. Эгиз, Труды Съезда по генетике, селекции и семеноводству, II (1930). [2] T. H. Goodspeed, Svensk Bot. Tidskrift, 17 (1923). [3] T. H. Goodspeed a. R. E. Clausen, Univ. Calif. Publ. Bot., II, No. 7 (1927). [4] L. E. Randolph, Proc. Nat., Acad. Sci., 18 (1932). [5] В. А. Рыбин, Труды Съезда по генетике, селекции и семеноводству, II (1930). [6] М. Ф. Терновский, Полиплоиды и гаплоиды при междувидовой гибридизации *Nicotiana* (1936).

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GENETICS

GALLERIA MELLONELLA L. AS A NEW MATERIAL IN GENETICAL INVESTIGATIONS

By V. P. OSTRJAKOVA-VARŠAVER

(Communicated by A. A. Sapehin, Member of the Ukrainian Academy, 13. III. 1937)

I. Some Data on the Biology, Morphology and Method of Breeding of *Galleria mellonella* L.

The data here published are the results of the first part of an investigation that we undertook for the purpose of studying *Galleria mellonella* as a new laboratory material for genetical investigations. *Galleria mellonella* has been used before as experimental material for laboratory investigations. We may mention the extensive series of works by Metallnikov ([1], [2]), devoted to the problem of immunity, the investigations of Manuilova and others ([3]) on the influence of density of population on the growth of caterpillars carried out with the silk moth and *Galleria mellonella*, and so forth. There have, however, been no attempts to study the latter as material for the genetical laboratory *.

In the imago stage the *Galleria mellonella* is a rather small moth, the females being a little larger than the males. In Fig. 1, *a* the female and male are shown in natural size. The female is of a uniform reddish-brown colour. The colouring of the male is more variegated. Both size of moth and colouring are characters which help the investigator to distinguish males from females. However, the best distinguishing character is the direction of the labial palpi; in the female they are directed forward and give the head a pointed form, while in the male they are directed upward and are covered with a frontal tuft of scales, so that the head has a blunt configuration. Seven to eight days is the duration of the life of the female. The male lives longer—two or three weeks—and is capalbe of mating during the whole period. The numerical ratio of the sexes is close to 1 ♀ : 1 ♂. The moth has an intense reproductive instinct and gives a very numerous progeny. Under our conditions of breeding them, a pair of moth gives on the average 400 individuals, while the number of eggs laid by a femal reaches 1000. The difference between the number of eggs laid and the number of individuals that reach adult state is due, first, to the death of a certain percentage of eggs, but secondly and principally, to the death of caterpillars owing to the competitions arising—between them on account of insufficient food supply and overcrowded con-

* When we were preparing our paper, Smith's article containing some data on the biology, genetics and method of breeding of *Galleria mellonella* was published.

tainers. Consequently, with sufficient food and spacious containers or the transference of one hatching to several containers, the number of offspring may be considerably increased.

The method we worked out of breeding *Galleria mellonella* under laboratory conditions is very simple. The culture are fed on honeycomb, the food of this moth, in glass jars having a diameter of 10 cm and a height of 24 cm. These jars are kept in the thermostat at a temperature of 30° C. This temperature,

Fig. 1. *a*—Male and Female of *Galleria mellonella*, Natural Size, *b*—Male and Female of *Galleria mellonella* of the «Miniature» Line.

according to our observation, is the optimal one. Under such conditions, the cycle of development lasts 5 to 6 weeks, the development of the egg lasting 7 to 8 days, the caterpillar stage lasting 21 to 24 days, and the chrysallis stage 10 to 12 days. Thus, under laboratory conditions it is possible to have 8 generations a year.

2. Natural Parthogenesis of *Galleria mellonella*.

The contradictoriness of the early data with respect to the presence of natural parthenogenesis in *Galleria mellonella* [Dönhoff ([5]), Tchang Yung Tai ([6,7]), Smith ([4])] and the interest which the phenomenon of parthenogenesis presents for genetical investigation led us to make a study of it.

We took individual cultures of virgin females. After fifteen days, in three of them, there were several caterpillars (1, 3 and 5) which developed subsequently into males and females. Inasmuch as all the conditions excluding the possibility of contamination and guaranteeing the virginity of the females were observed in the experiment, there is every reason to consider that these individuals were the results of natural parthenogenesis. Thus, the percentage of cultures giving parthenogenetical individuals under the given conditions was 3 ± 1.705 per cent. Nothing can be said regarding the percentage of hatched eggs since we did not count the eggs in these cultures. Still, there is every ground for supposing that it was very small.

A comparison of our results with the data of Smith reveals a disagreement in both the percentage of egg batches in which parthenogenetic caterpillars appeared (3 and 10%) and in the length of development of the unfertilized eggs. We are inclined to believe that the difference is due to different breeding conditions, about which, unfortunately, Smith's paper is silent.

3. Mutations in *Galleria mellonella* with Respect to Size of Wings

Among the offspring of the fourth generation of inbreeding (brother and sister), we found two moths, a male and a female, which had diminished wings, both fore and back, as compared with the normal. Crossed with each other, they gave offspring that were also short-winged. The hereditability of this variation was thus discovered. Examination of the changed wings showed that venation and shape of wings remained unaltered. The wings were changed only in length and width—that is they were miniatures of the normal wing. A special genetical analysis of this mutation established its recessive and autosomal character and its monohybrid type of inheritance. We named this mutation the «miniature». A male and female of the «miniature» line are shown in Fig. 1. *b*.

a *b*

Fig. 2. Equatorial plates of spermatocytes of the I (*a*) and II (*b*) orders.

4. Karyotype of the Males of *Galleria mellonella*.

Investigation of the karyotype of the males was carried out on material from the testes. On the basis of a microscopic study of the metaphase plates of spermatocytes of the order I, a haploid chromosome number was established (=30). No variation was noted in the chromosome number. These results are in good agreement with the data of Kemnitz([8]). In the metaphase plates of the spermatocytes as in the majority of the moths, the chromosomes are spheroidal, vary somewhat in size and are distributed uniformly at some distance from each other. The equatorial plates of spermatocytes of the I (*a*) and II (*b*) orders are shown in Fig. 2.

Our data would seem to characterize *Galleria mellonella* as quite convenient and interesting subject for genetical investigation. Its great prolificity, the ease of breeding it, the possibility of obtaining offspring all the year round and the comparatively short life cycle—all these characteristics are to the advantage of *Galleria mellonella* as compared with such genetical material from the *Lepidoptera* (as *Limantria dispar and Bombyx mori*). At present, of course, it is still impossible definitely to state all the problems of genetics for the solution of which *Galleria mellonella* may be used as experimental material. However, it may be supposed that among these are the problem of artificially obtaining mutations, the investigation of quantitative characters, parthenogenesis, and so forth. The existence of another species, *Achroea grisella*, will probably offer a possibility of interspecific hybridization. The wide distribution of *Galleria mellonella* throughout the world and, together with this, the existence of geographical strains of it make it an interesting material for geno-geographical investigations as well as for studies of intersexes, carried on at present principally on *Limantria dispar*.

Together with this, the introduction of such a subject into the laboratory would make it possible to hasten the solution of certain standing problems in the theoretical and practical genetics and selection of the silk moth.

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REFERENCES

[1] S. Metalnikov, Zentralbl. f. Bacter., **41** (1906). [7] S. Metalnikov, Archives de Zool. Experimentale, **12** (1908). [3] Н. А. Мануилова, Н. П. Кузьмина, В. В. Алпатов, Журн. соц. шелководства, № 3 (1931). [4] Th. Smith, Record of the Genetics Soc. of Amer., **4** (1935). [5] Dönhoff, Bienen Zeitung, XIV (1858). [6] Tchang Yung Tai, Comptes Rendus de la Société de Biologie, II (1925). [7] Tchang Yung Tai, Comptes Rendus de la Société de Biologie, **103** (1930). [8] Kemnitz, Arch. Zellforsch., **12** (1914).

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PLANT PHYSIOLOGY

FORMATION OF CITRIC ACID IN THE MAKHORKA LEAF (*NICOTIANA RUSTICA L.*)

By O. J. SOBOLEVSKAJA and V. S. BUTKEVIČ (BUTKEWITSCH), Corresponding Member of the Academy

New possibilities of using makhorka not only for smoking but also as raw material for the production of nicotine and citric acid ([10]) have arisen in our Union during these last years.

According to literary data, the content of citric acid in the makhorka leaf may be greatly increased during the process of its post-harvest treatment, i. e. during its drying ([3, 6, 9]). It has been found, however, that the content of citric acid in the makhorka leaf varies a great deal. Under some conditions of drying, the increase in its content exceeded 50% of the original quantity. Under other conditions not only no increase could be detected but there was observed a decrease reaching sometimes as much as 30% ([3]). Such variations show that some conditions of drying are more favourable to the production and accumulation of acid, while under others its consumption predominates.

During the last two years, the Institutes of Tobacco and Makhorka Industry concentrated their attention upon the conditions of leaf drying leading to an accumulation of citric acid. However, these conditions have not yet been definitively determined. The methods of makhorka cultivation allowing to regulate the relative content of carbohydrates and nitrogenous substances in the leaves have been worked out in more detail. It is possible to modify the content of these substances in the makhorka leaf by different methods of «topping» and by using different fertilizers.

However, we cannot yet determine which of the two basic groups of substances is more desirable in growing makhorka for the production of citric acid.

It has been shown ([1]) that carbohydrates are the source of citric acid in the biochemical metabolism of lower organisms. But the question of the origin of citric acid in higher plants has not yet been solved. Although most authors consider that it originates from carbohydrates, some still connect its formation with the conversion of protein substances. The present article will deal with the chemical processes of citric acid formation in the makhorka leaf.

As material for experiments we used the leaves of makhorka (*N. rustica*) of the Kudriavaya variety; the plants were grown in pots. The seeds were received from the All-Union Institute of Tobacco and Makhorka Industry.

They were of the 1935 crop. For our experiments we took leaves grown under the conditions of a relative increase of the content of carbohydrates and a decrease of the content of protein substances. This was effected by late and high topping. In general, the aim of topping was to increase the content of carbohydrates in the leaf, and late topping was recurred to in order not to retard the flow of nitrogenous substances in the direction of the reproductive organs which takes place mostly during the first period of flowering. A part of the experiments was conducted with leaves of normal (non-topped) plants, in order to determine the importance of the operation of topping.

The leaves were taken always from the same whorls in the second half of the day, i. e. during the period of the greatest accumulation of carbohydrates. The leaves were washed with tap water and dried between sheets of filter paper; they were then cut in two along the midrib. One half was used for the experiment, while the other served for determining the initial quantity of citric acid. We usually took six half-leaves weighing about 60 g. Prior to experiments, the leaves were exposed to light for a few minutes so that their stomata might open. After that they were subjected to a vacuum injection (4) of a sterilized solution of glucose (0.1 mole), washed again with tap water, dried between sheets of filter paper, weighed for a second time, in order to determine the amount of glucose introduced, and then hung up in the humid chamber. In some cases, the control leaves were injected with sterilized tap water and placed in the same chamber; in others, they were simply hung up in the humid chamber without being previously injected with water. The leaves were kept in the chamber for three days. Then they were taken down, killed by high temperature (90°) during 15 minutes and subjected to further treatment.

Citric acid was determined in the form of pentabromacetone according to a combined method [Hartmann and Hillig (7), Piatnizkij (5), Kotnizkij and Bogatyrchuk (2)].

The figures of the analyses are referred to the initial raw weight of the leaves. The results of the experiments are shown in Tables 1 and 2.

Table 1

Material used	Content of citric acid in % of raw weight of leaf			Increase of citric acid in % of its initial quantity	
	In the initial material	In the experiment (injection of glucose)	In the control	In the experiment	In the control
Young leaf	0.173 0.168	0.368 0.378		+119.41	
	0.188 0.192		0.229 0.223		+18.94
Technically ripe leaf	0.401 0.396	1.036 1.028		+159.29	
	0.383 0.388		0.725 0.730		+89.35

Table 1 shows that the young leaf contains considerably less citric acid than the technically ripe one. When kept in the humid chamber in the control, the former also accumulates considerably less acid than the latter. The

same relation holds after the injection of glucose, but the differences are less pronounced, which seems to be due to an equalization of the contents of sugars in the leaf as a result of the injection. Thus, the young leaf is as able to accumulate citric acid as the technically ripe one and the low content of acid in the young vegetating leaf seems to be due to an intenser metabolism.

The same relations were observed in the experiments with leaves of a fruit-bearing and a topped plant (Table 2).

Table 2

Material used	Content of citric acid in % of raw weight of leaf			Increase of citirc acid in % of its initial quantity	
	In the initial material	In the experiment (injection of glucose)	In the control	In the experiment	In the control
Technically ripe leaf from fruit-bearing plant	0.227 0.229	0.586 0.570		+153.5	
	0.220 0.215		0.286 0.278		+25.34
Technically ripe leaf from a topped plants	0.401 0.396	1.036 1.028		+159.29	
	0.383 0.388		0.725 0.733		+89.35
	0.494 0.501	0.957 0.946		+ 91.34	
	0.570 0.571		0.844* 0.899		+52.80

* The control was injected with water.

All the experiments with leaves injected with glucose and kept in the humid chamber show that the increase of citric acid content in the technically ripe leaf is about 50% greater than in the control leaf.

In the case of leaves possessing a smaller amount of substances serving as material for the formation of citric acid (young leaf and leaf of a fruit-bearing plant), the increase of its content after injection is about twice as much as in the control (Tables 1 and 2). All these data show that the makhorka leaf, independently of its age, reacts positively, in respect of the accumulation of citric acid, to the introduction of glucose.

Besides the experiments described above, we carried out an experiment in which parallel calculations of the consumption of sugars and the increase of citric acid content were made. This experiment showed that the increase of citric acid content is accompanied by a suitable consumption of sugars (Table 3).

The following conclusions may be drawn from the data cited by us.

As shown by the experiments the sugars are the materials out of which citric acid forms in the makhorka leaf. This follows from the increased accumulation of citric acid in the leaves when they are injected with glucose solution as well as from the relation between the increase of citric acid content in the leaf and the consumption of sugars.

Table 3

Content of Citric Acid and Sugars in the Makhorka Leaf in Percentage of Initial Raw Material

	Before the experiment	After the experiment	Consumption of sugars	Increase of citric acid content
Reducing sugars	1.367	0.273	1.094	
Citric acid	0.415	1.032		0.617

When working out the conditions of post-harvest treatment of makhorka as raw material for the production of citric acid, it should be remembered that, by keeping the leaves in humid atmosphere, a considerable increase of citric acid content (up to 100% of its initial quantity) may be effected.

Inasmuch as carbohydrates seem to be the source of citric acid in makhorka leaves, it is essential, in growing makhorka for the production of citric acid to obtain leaves with the highest possible content of carbohydrates. For this purpose, suitable methods of makhorka cultivation should be applied and attention should be paid to the time of harvesting.

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REFERENCES

[1] Butkewitsch, Jahrb. f. Wissensch. Bot., **64** (1925); Микробиология (1932). [2] Котницкий и Богатырчук, Тр. ВИМП (1935). [3] Максимович и Пигальская, Сб. работ по изучению послеурожайной обработки махорки ВИМП (1935). [4] Móthes, Ber. d. D. Bot. Ges., **46** (1928); General Vers., H. 59. [5] Пятницкий, Тр. ГИТ, вып. 81 (1931). [6] Смирнов, Табаковедение, ВИТП (1933). [7] Hartman a. Hillig, Journ. of the Assoc. of Offic. Agric. Chem., **17**, № 3 (1934). [8] Чубуков, Агротехника табака (1932). [9] Шергин, Сб. ВИТИМа, 118 (1935). [10] Шмук, Сб. ВИТИМа, 102 (1933).

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